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Keywords: Neolithic revolution

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The emergence of government as organized violence-cum-robbery[*]

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Abstract

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Keywords: government, emergence of hierarchy, monopoly of violence, Neolithic Revolution.

JEL codes: H10, O43, N4, P48.

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1 Introduction

One of the biggest questions is that of the origins and emergence of hierarchical forms of 'government' in human societies. The archaeological record shows that groups with enduring hierarchical power relations among adult males, based on differences in the accepted capacity for violence, and accompanying inequality in wealth, emerged sometime after human habitation and agricultural production had become spatially fixed. The latter is associated with the complex of developments which is called in shorthand the Neolithic Revolution.

The hierarchical societies that developed over the millennia following the Neolithic Revolution were remarkably similar (Diamond, 1997, 268–89; Morris, 2014, 88; Borcan, Olsson, and Putterman, 2021). They shared four clear characteristics: (i) monopolized coordination and control of violence, (ii) individual specialization in different roles, (iii) stratification based on these roles, and (iv) demarcation and inheritability of private possession and increasing material inequality (Carneiro, 1998; Kohler et al., 2017). In essence, these comprise the defining characteristics of government as they are commonly understood (Diamond, 1997, 273–8; Sánchez de la Sierra, 2020). These subsequently developed in more complex societies, for instance with further distinctions of roles, emerging bureaucratic organizations, and impersonal relations embedded in formalized procedures (Mann, 1984; Tilly, 1985; North, Wallis, and Weingast, 2009). We will use the term 'state' for these more developed societies while referring to the early hierarchical societies as 'chiefdoms', and to the general structure of political authority as 'government'.[1]

In this paper, we present a game theoretic model in the spirit of the approaches by Binmore (1994) and Myerson (2008, 2015), to analyze the distinctive requirements for the emergence of 'government', as exemplified by the difference between tribal societies and chiefdoms, the two ideal-typical forms of human organization on either side of its emergence. Throughout the paper, we use the terms 'tribe' and 'chiefdom' as ideal types, while acknowledging their diversity and context-specificity in real-world settings. Also, we acknowledge the non-linearity of their development. Recent research has abundantly shown how non-hierarchical societies existed alongside hierarchical ones for long, and often resisted incorporation, and how hierarchical societies frequently disintegrated or reverted to non-hierarchical ones (Feinman, 2013, 2017; Scott, 2017). The logic of these non-linear processes will be shown throughout this paper.

The ideal-typical tribe did not have pronounced hierarchies and had norms

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[1] Note that this distinction between 'state' and 'government' is not always made in the relevant literature. For instance, Allen, Bertazzini, and Heldring (2023) use the two interchangeably, and while stating to look at the origins of government, they analyze the rise of states of the more advanced type, as appears from the indicators they use (palaces, administrative buildings, and bureaucracies).

that turned the group against differences in violence capacity and coercive force (Service, 1962; Earle, 1978; Creamer and Haas, 1985; Carneiro, 1998; Acemoglu and Robinson, 2019). When differences in status occurred, these were ephemeral, context-specific and temporary in nature. This even applied to a so-called 'big man', found in some tribes, whose position was gained in action or war, but was not formalized, ascribed or permanent, let alone inheritable. His role was agenda-setting; he had no means of enforcing decisions upon the group. Hunter-gatherer societies typically acted against those striving to dominate and they imposed all kinds of sanctions against such behavior (Scheidel, 2018, 28–32; Wrangham, 2019, 158–67). Tribes in this ideal-typical form thus did not possess any of the four key characteristics highlighted above.

Chiefdoms, however, possessed all four of these characteristics, at least in a rudimentary form. This type of social organization, emerging first around 5500 BC in the Fertile Crescent, typically had (i) some form of hierarchy, (ii) with a permanent chief, (iii) some form of monopoly of violence in the hands of this chief and a group of violence specialists under his command, who could enforce his decisions, and (iv) some form of surplus extraction by way of tribute and some form of property rights (Earle, 1978; Diamond, 1997, 268–77; Carneiro, 2017). To put it sharply: the first steps towards the four key characteristics of government and social organization as we know them now, were taken with the formation of chiefdoms. These first steps form the central focus of this paper.

Chiefdoms and hierarchical forms of government emerged in six out of seven locations where the Neolithic Revolution occurred earliest and independently from each other. This did not happen in New Guinea but it did in Mesopotamia, the Nile Valley, the Indus valley, the Yangtse and Yellow River valleys in China, Mexico and Peru (Spencer, 2010; Borcan et al., 2021). Even though these locations did hardly have mutual contact and were hardly or not at all influenced by each other, all six of them developed similar social and institutional characteristics.

Even though a link to the Neolithic Revolution is obvious, and in some economics papers the rise of hierarchy is seen as a logical solution to collective actions problems (Allen, Bertazzini, and Heldring, 2023; Dal Bó, Hernández-Lagos, and Mazzuca, 2022), recent work shows that the introduction of agriculture was not a sufficient cause for hierarchy to emerge. Instead, it occurred only with a time lag of several millennia following sedentism and agriculture, and its rise was not at all natural or automatic (Scott, 1998, 2017). First, even though the small elite was clearly better off in chiefdoms and later states than they would have been in tribal societies, the great majority of the people seems to have been worse off (Bogaard, Fochesato, and Bowles, 2019). Forms of coercion and slavery were introduced, first small-scale and later more intense and widespread, while research on skeletons shows that people had less varied diets, were less healthy, and lived shorter (Cohen and Crane-Kramer, 2007). The transition was not a Pareto improvement and, therefore, unlikely to have been based on consensual decision-making. Second, it has become clear that the rise of government and social differentiation was not the automatic, direct result of the Neolithic Revolution and the productivity rises it brought about (Gurven et al., 2010). While physical output increased, labor productivity may not have risen at all during the Neolithic Revolution. More specifically, the appropriability of produce (that is, the spread of the cultivation of grain that can be easily collected, transported and stored) played a larger part in social differentiation than any possible productivity rise (Bowles and Choi, 2019; Mayshar, Moav, and Pascali, 2022). The lack of straightforwardness in the emergence of government is reflected in the fact that in all of these six locations it took some five millennia from the beginnings of agriculture to its protracted start. In addition, tribal societies survived alongside chiefdoms for long and, also, chiefdoms frequently dissolved again. Clearly, tribes were not easily absorbed in chiefdoms and were not keen on being absorbed at all (Scott, 2017).

The emergence of hierarchy is, therefore, something that needs to be explained. Accordingly, we develop a game-theoretic model to scrutinize the critical differences between tribes and chiefdoms, thus to explain the emergence of hierarchical societies, or ‘government’. The core ingredients of the model are (i) a large number of (ex ante) homogeneous players competing over productive and appropriative actions, (ii) endogenous private monitoring, and (iii) the possibility of transferring produced resources. While there is an over-arching repeated game encompassing all equilibria that we study, the complete game is most easily explained by developing it in three steps as some aspects of the game will only be relevant for more elaborate equilibria. Interestingly, the use of these more complicated features roughly coincides with the evolution of the institutions observed in human societies. The first two steps do not use the repeated game structure, so we can concentrate on the stage game.

The stage game of the first step consists of only two decision phases, where all players move simultaneously. In the first investment phase, players decide how to divide an initial endowment between investments in edible resources and violence capacity (‘arms’). As a default, players do not observe the investments of other players. Edible investments come in two types: ‘grain’, which is more valuable but fully contestable; and ‘tubers’, which are less valuable but non-contestable. In the second robbery phase, players choose whether or not to attack another player, and if so, which player. A successful attack robs part of the attacked player’s grain.

The first best outcome of this game is therefore that all players invest their full endowment in grain, since any investment in violence capacity or in low productive tubers comes at the expense of total output. This outcome is not an equilibrium, however, since if all players do this, a deviant would be better off by investing in arms. We show that there exists an 'anarchy-like equilibrium', where all players similarly divide their endowment between grain, tubers and arms. Since all players invest equally in arms and since an attack on an equally armed player fails, the only chance for a player to rob another player is by teaming up with a third player. Since all players decide randomly who to attack, this can happen only by chance.

The second step adds a crucial element by the introduction of endogenous private monitoring. In the investment phase, players can invest part of their endowment in monitoring other players. By monitoring another player, the investor observes this player's investments in the investment phase before deciding who to attack in the robbery phase. Using this element, we demonstrate the existence of a 'tribal equilibrium', where all players monitor everybody else, invest just a small amount in arms and the remaining endowment in grain. Though investment in monitoring comes at the expense of aggregate output (in a first-best world, no resources would be spent on monitoring), it helps containing violence by collective punishment of deviations. In equilibrium, there is no robbery and all players receive the same payoff, corresponding to the equitable situation within tribes. This requires tribes to be neither too small nor too large. If a tribe is too small, the individual investment in arms required for an effective collective deterrence is too big. If a tribe is too large, the cost of monitoring all other tribesmen is too high. Contrary to the anarchy equilibrium, the tribal equilibrium requires social norms, which are endogenously enforced, or more particularly: beliefs about how players are expected to behave and how other players should and will respond when one player deviates from these norms.

The third step introduces two additional features. First, it allows for two additional phases of 'voluntary' grain-transfers between players in between the investment and robbery phase of the stage game. Second, it introduces a repeated game structure where the stage game is repeated infinitely. Transfers are a natural extension to consider in the context of the gradual expansion of cultivation of grains by increasingly sedentary societies, post-Neolithic, and the growing amounts of storable and transferable grain. This extension allows for asymmetric and hierarchical behavior. The first phase of transfers (dubbed 'taxation') is a more efficient form of extraction than outright robbery as in the anarchy equilibrium. These transfers do not rely on the actual use of violence but on the threat of using it, allowing a relatively limited investment in violence capacity by a few 'soldiers' to be used to enforce transfers from a large group of 'farmers'. The latter are obliged to pay taxes and are not allowed to invest in arms. Deviations are punished by attacks on the deviant by a coalition of 'soldiers'. Hence, though the threat of using violence is omnipresent, the actual use of violence is negligible. This social norm introduces a monopoly of violence in the hands of a dominant coalition (North et al., 2009). These tax-transfers relax the budget constraint of the 'chief' (the beneficiary of these transfers) beyond his initial endowment. He uses this relaxation in the second phase of transfers (dubbed 'wages') to buy soldiers' loyalty to enforce these tax-transfers by the threat of using violence. However, farmers will only invest in grain and pay these transfers if the use of violence is contained; if not, they will protect themselves against robbery by investing their full endowment in non-contestable tubers. Soldiers therefore have to commit not to abuse their violence capacity. The only way they can do so is by using the repeated game structure. This equilibrium has to deal with the threat of 'palace revolts', where a coalition of soldiers robs the tax-receipts of the chief.

This chiefdom equilibrium contains three different 'social roles', assigned by nature in a pre-phase to the game and perfectly observable to all players—many farmers, a few soldiers and one chief—who receive widely different payoffs in equilibrium. Though these roles do not change the capacities or physical constraints on players' strategies, this role-attribution is self-enforcing. Conditional on the strategies of other players, no player is better off by attempting to play another role than has been attributed to him, even though some roles have much larger payoffs than others, since this deviation will be adequately punished.

Although chiefdoms resemble models wherein chiefs offer protection in exchange for payments—in traditional 'stationary bandit' fashion (e.g., Olson, 1993)—the implications of our are different. First, in our model, tribes are also capable of containing violence. Moreover, the protection offered by chiefs is strictly payoff-decreasing for the common member of the chiefdom. In contrast, soldiers and in particular the chief receive much higher payoffs than attainable in tribes.

A strength of our model is that it enables us to simultaneously analyze conflict within and between groups within a single game. Accordingly, we study competition between two groups—first, between two tribes, then between a tribe and a chiefdom—in a further extension. The repeated game allows for peaceful equilibria when relying on long-term play. The competition between two tribes yields a mixed strategy equilibrium wherein tribes randomize between attacking the other tribe and not doing so. The competition between tribe and chiefdom yields an equilibrium wherein the chiefdom dominates through better coordination of violence. This is despite the relative vulnerability of chiefdoms deriving from the concentration of surpluses of grain in the hands of the chief and the attraction of appropriating them. Tribes no longer require internal monitoring when they are in violent conflict with other groups, due to a common enemy-effect, which implies that they can remain substantial threats to chiefdoms.

The three core ingredients of the model—a large number of players, endogenous private monitoring, and transfers—allow us to study cooperative social arrangements of both egalitarian and hierarchical form, without presupposing the aggregation of resources in a common pool or ex ante differences between players. Here we deviate from many of the canonical political economy models (including Acemoglu and Robinson, 2006; Besley and Persson, 2011).[2] Furthermore, we emphasize the short-term effects of the information structure, personal—thus limited—violence capacities, and transfers on the incentive structure of the game. Hence, rather than relying on long-term relational dynamics to support cooperative play, our approach highlights the institutional frictions and solutions required to support cooperative egalitarian equilibria, and coalitional play within the dominant group of hierarchical equilibria.

In our model, inequality emerges only ex post, as an equilibrium outcome of the game. Coalitions exist in both the egalitarian and hierarchical form and crucially depend on investments in monitoring and violence. The combination of endogenous private monitoring and personal violence implies that a small group of soldiers and the chief—the dominant coalition in the terminology of North et al. (2009)—can force other players into submission, when they can credibly commit to (i) punish these other players for investing in arms and (ii) punish deviant behavior within the dominant coalition. Then, the non-monitoring players cannot effectively coordinate strategies whereas the chief who does monitor everybody else can. The internal commitment problem of punishing those who abuse their power versus farmers is solved by high equilibrium rewards and an information structure wherein soldiers cannot easily collude against the chief. Hence, even within the coalition, it is the monopolization of information that lends the chief his edge over other, more fully armed players. This is also the main efficiency advantage of chiefdoms: whereas everybody must monitor everybody else to sustain a tribal equilibrium, only the chief has to do so in the chiefdom equilibrium. When population sizes increase, this efficiency-benefit gives chiefdoms an edge over tribal societies.

With this model we aim to further our understanding of the shift from tribal societies to large, consolidated chiefdoms and to contribute to the question of how these societal orders can be stable and with what distribution and aggregate size of payoffs. Chronologically, this concerns the period from the Neolithic Revolution to the consolidation of chiefdoms, a period roughly spanning three to five millennia. The sequence of equilibria helps to understand the protracted emergence of chiefdoms, underlining the success of tribes and the institutional innovations required for chiefdoms, as well as the vulnerability of these chiefdoms to coordinated attacks by

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[2] See also Grossman and Kim (1995, 1996); Mayshar et al. (2017); and Dal Bó et al. (2022)—and for models wherein hierarchies revolve around pooled resources—e.g., Skaperdas (1992); Hirshleifer (1995), and for relevant applications Konrad and Skaperdas (2012); van Besouw, Ansink, and van Bavel (2016). Compared to the large repeated-games literature on cooperative social equilibria, our emphasis is on the involved stage game of our model rather than enforcement of cooperative play in the long run—see, for instance, Ali and Miller (2016); Acemoglu and Wolitzky (2020)—or that, similar to the previously mentioned papers, build on ex-ante differences or a small number of players—as in Aldashev and Zanarone (2017); Greif (1993); Greif, Milgrom, and Weingast (1994); Milgrom, North, and Weingast (1990); Myerson (2008, 2015).

tribes and the low payoffs of most players in chiefdoms.

This paper relates to a growing literature on the emergence of government (Acemoglu and Robinson, 2019; Sánchez de la Sierra, 2020; Allen et al., 2023; Dal Bó et al., 2022) and, particularly, in the context of pristine emergence of social hierarchies (Baker, 2003; Dow and Reed, 2013; Bowles and Choi, 2019; Borcan et al., 2021; Mayshar et al., 2022). Our model contributes to understanding the protracted emergence of chiefdoms and the long-term coexistence of egalitarian societies alongside more hierarchical ones. Like several influential contributions to this literature, we emphasize the emergence of a coalition of players who establish a de facto monopoly of violence (e.g., North et al., 2009; Konrad and Skaperdas, 2012; Acemoglu, Egorov, and Sonin, 2012; Acemoglu, Robinson, and Santos, 2013; Naidu, Robinson, and Young. However, we emphasize that this dominant coalition hardly serves an economic purpose: the coalition does not provide additional protection in exchange for taxation, it simply imposes taxation under the threat of violence. While we do not deny that governments and later state bureaucracies can generate public goods and solve coordination problems (Olson, 1993; Grossman, 2002; Konrad and Skaperdas, 2012) and this could even drive and legitimize the formation of state organizations (Allen et al., 2023), we show that the original rise of hierarchical government was not demand-driven or based on mutual consent. Instead, our model supports the literature stressing the extractive origins of government (Sánchez de la Sierra, 2020; Mayshar et al., 2022).

Our emphasis on access to and control over information as the central ingredient of power connects our work to other models of information within hierarchical forms of government (Myerson, 2008, 2015; Egorov and Sonin, 2011; Mayshar, Moav, and Neeman, 2017). Whereas these models focus on exogenously determined information structures and long-term interactions, our results derive from endogenous private monitoring within the stage game. We show that, given individual capacity constraints in investments and use of violence, the monopolization of information is the crucial component underpinning hierarchy. In addition, the information structure allows chiefdoms to grow substantially larger than tribes, while centralizing information within the dominant coalition ensures stability in the sense that heavily armed members of the coalition are less capable of threatening the chief. Our emphasis on information is in line with statements in some of the ethnographic and anthropological literature and with the evidence on the role of chiefs more generally—see Section 2 and Earle (1978); Creamer and Haas (1985); Carneiro (1998).

Finally, our model is related to a large literature on the institutions on which different forms of government are predicated (Acemoglu and Robinson, 2006, 2019; North et al., 2009; Besley and Persson, 2011). Whereas tribes rely on collectively enforced retaliation against deviants, with free-riding concerns being solved by full monitoring—related to the literature studying private-order institutions (Milgrom, North, and Weingast, 1990; Kandori, 1992; Greif, 1993; Greif, Milgrom, and Weingast, 1994; Dixit, 2003)—chiefdoms require several institutional innovations. One of them is the demarcation of property, allowing for systematic payments of wages and taxes over prolonged periods of time (North et al., 2009; Bowles and Choi, 2019). This, however, can only be enforced by control over information. Jointly, monopolizing information and the demarcation of large rents extracted from a wider population, provide the solution for commitment problems among the coalition of chief and soldiers. These players cannot be controlled by the threat of violence in the absence of information constraints, nor can they commit to abstain from abusing players outside the coalition. Nonetheless, the information and property extraction structures of chiefdoms generate rents that are so large that they can keep members of the coalition in check—much in line with the discussion by North et al. (2009) of rent extraction as foundation for stability in natural states.

The remainder of the paper is organized as follows. In Section 2, we use archaeologic, ethnographic, and historical literature to offer further contextualization and sharpening of our model. In Section 3, we set up a model based on the premises set out in Section 2. We analyze the core, single group equilibria—tribe and chiefdom—in Sections 4 and 5. In Section 6, we analyze equilibria of two competing groups; either two tribes or a tribe and a chiefdom. In Section 7, we confront the hypotheses on the shift from tribes to chiefdoms with the logic of our model and in Section 8 we discuss our main findings and their implications.

2 Historical Background

For long, the prevalent form of social organization of humans has been the band or tribe of hunter-gatherers. Chiefdoms emerged first around 5500 BC in the Fertile Crescent and they sprung up in the following millennia all around the globe—such as in China around 4000 BC, Britain around 3000 BC, and Mexico and Peru around 1500 BC (Diamond, 1997, 362–3; Carneiro, 2017). Alongside the growing numbers of chiefdoms, bands and tribes of hunter-gatherers still existed, but they gradually and intermittently lost terrain to the larger chiefdoms and were pushed to areas less suitable for intensive agriculture. Important to note is the long time-lapse of several millennia between the emergence of sedentism and agriculture on the one hand and that of chiefdoms on the other.

The same areas, southern Mesopotamia in particular, saw the development of small states from about 3500 BC, with further state formation in the following centuries, and diffusion from its original centers. This process is documented across the Fertile Crescent and the Nile Valley, in the Indus valley around 2600 BC, the

Yellow River around 2000 BC, and much later in Mexico and Peru (Borcan et al., 2021). In all these cases, emerging states eventually overtook existing chiefdoms, often developing out of one of these (Spencer, 2010). It is important here to note, however, that the development of a monopoly of violence in the hands of a specialized elite, which used this monopoly to extract surpluses and defend property rights, was already apparent with the rise of chiefdoms. Even though further sophistication took place in later states, and government in many instances developed more functions, including beneficial ones to society, the emergence of chiefdoms thus formed the most momentous step in the development towards government and social organization as we know it now.

Indeed, while acknowledging the diversity among chiefdoms, as an ideal type they share clear characteristics that separate them from tribes (Carneiro, 1998). Chiefdoms surmount families and villages and integrate them into a political unit under the permanent control of a paramount chief. The position of the chief is ascribed and hereditary. He monitors the people in his chiefdom, he keeps track of taxes paid and controls whether people acquire arms. The chief possesses a kind of information monopoly, particularly for critical information, based on which he can make decisions. Some scholars have even considered this the main function of the chief, as an 'information processor' whose power over the chiefdom is based on monitoring and controlling information (Earle, 1978; Creamer and Haas, 1985, 740). Around this chief, and under his command, a group or coalition of violence specialists is formed—varying from a small group of dozens of close supporters of the chief in simpler chiefdoms to a larger class of a few hundred in the more complex ones (Gat, 2008, 211–2 and 228). Helped by his control over the means of violence, the chief extracts tribute and taxes from members of the chiefdom, typically farmers, with the revenues being partly redistributed to his military retinue and partly retained by him, enabling him to become wealthier than other people within the chiefdom. The position of the chief is buttressed not only by his command over the group of soldiers and the associated coercion he can exert under the threat of violence, but also by his ritualistic or religious role and norms underpinning the legitimacy of his position. Supernatural beliefs were increasingly employed to justify hierarchy and extraction (Gat, 2008, 223–8; Diamond, 1997, 277–8; Bentzen and Gokmen, 2023). This normative framework further reduced the need to resort to open violence within the chiefdom.

One may be tempted to think that the emergence of this set of characteristics was a logical or natural process. At least three observations support this line of thinking. First, the fact that this set has become ever more dominant in the course of history, which lends a teleological flavor to its development. Second, there is the similarity of its—often independent—emergence in separated places around the globe, which points to a common, underlying logic in this set of characteristics. Third, there is the observation that its development went along with a rise of productivity and growing population densities, especially during and following the Neolithic Revolution, and an associated growth in the size of social units. While the bands or tribes of hunter-gatherers counted a few dozen to maximum a few hundred people, and relied on face-to-face interaction, chiefdoms could extend to thousands of people. Within these chiefdoms, face-to-face contact was insufficient to contain internal violence and robbery. With the rise of group size, it became necessary to develop a more complex social organization. Moreover, the same chiefdoms continuously had to deal with the external threats emanating from competing chiefdoms and hunter-gatherer bands and tribes, and their acts of violence and robbery.

The latter already points to the overriding importance of violence; containing violence is the greatest challenge any society is facing (North et al., 2009). Within tribes, and helped by their small size, personal relationships and contacts were the main mechanisms to contain violence. Between tribes, two ideal-typical situations can be envisaged (Diamond, 1997, 2012; Gat, 2008). The first is a peaceful one, or rather one of deterrence, where those who attack members of another tribe are withheld and punished by the members of his own tribe. The second is a violent one, where tribes are mutually attacking and raiding each other, and warfare could even be endemic. Still, this warfare was rather unorganized, with the 'big men' leading the tribe rather by heroic example than having any disciplinary power (Gat, 2008, 183–9; Henrich, Heine, and Norenzayan, 2010).

The rise in the scale of social units sketched above went along with the emergence of arrangements to coordinate violence and to contain open violence (North et al., 2009, 51–5), most notably the development of institutionally underpinned hierarchies, varying from more small-scale and simple ones in the chiefdoms to more large-scale and complex ones in states. As observed, on the one hand these hierarchies can be seen as a logical response to the growing size of societal units, making it impossible to contain open violence by way of face-to-face interaction, but causality has likely also worked the other way around, the driving force being the desire of a minority of society to monopolize violence capacity and use this to coerce the great majority and extract their surpluses. As this extraction happened within ever larger societal units, this minority was able to become ever wealthier (Kohler et al., 2017; Scheidel, 2018, 33–61). Asymmetric capacity for violence forms an essential part of the constitution of such hierarchies. This asymmetry leads to coercion but would be much more destructive without such order that informs, structures, and limits the use of violence.

In chiefdoms, the violence specialists form a coalition, headed and commanded by the chief; a constellation that enables them to extract surpluses and makes them better off than ordinary farmers and at the same time limits the use of open violence. The need to resort to open violence was additionally reduced by ideology and social or religious norms, and the associated forms of punishment, like excommunication, damnation or ostracization, which can also be considered a type of instruments of violence. Moreover, these norms legitimize the use of open violence by the coalition of violence specialists, or the government they formed, against those who break them, thus making violence more effective.

The formation of government can therefore be seen as a crucial way to deal with the problems of violence and appropriation in larger scale societies. However, even though the older literature has sometimes seen this as an obvious development (Skaperdas, 1992; Olson, 1993), it is clearly not one of linear and unilateral progress. Bands of hunter-gatherers and tribes, which also had the capacity to contain violence, albeit in a different way, continued to exist alongside chiefdoms and even states, and they were not keen on being absorbed by them. In many cases, they could be coerced only through the use of force, subjected and incorporated in chiefdoms and, later, states (Scott, 2017). Rather than a voluntary, joint decision of a populace, it was frequent violence and warfare (Carneiro, 1998), and coercion under the threat of violence and appropriation by an emerging elite (Mayshar et al., 2022), which were key to the formation of larger, more hierarchical societies. The shift of tribe to chiefdom and to kingdom was a very slow and protracted one, fraught with coercion under the threat of violence. This is the process—with the emergence of chiefdoms as the most momentous step—our model aims to elucidate.

In order to contextualize our model, we can build on the archaeological, anthropological, ethnographic and historical record, which helps tracing changes of violence and material life during the shifts from tribes to chiefdoms.

First, the effects on the incidence of open violence have been mixed. There is broad agreement that open violence has declined in the very long run, that is: in the long period leading up to modern states (Wrangham, 2019, 235–47). Also, it is clear that high rates of violent deaths were found in tribal societies in the modern period, mainly evidenced by ethnographic data, as for nineteenth century Australia and twentieth-century New Guinea and Polynesia (Gat, 2015; Wrangham, 2019, 13–9). The evidence for prehistoric tribes, however, is more mixed. Some scholars argue that the risk of violent death was very high there, citing figures of 10–40 per cent of adults showing clear traces of violent trauma, including traces of projectiles and cranial vault fractures (Gat, 2015). Other scholars are more nuanced (Ferguson, 2013), while some recent studies even argue that rates of death caused by violence in prehistoric tribal societies were not out of the ordinary for the human species at all (Gómez et al., 2016). While assessments of violence among prehistoric tribes may differ, and we do not take a position on this, there seems to be scholarly agreement that violence and warfare have become more frequent and intense in periods of transition from tribal societies to chiefdoms (Carneiro, 1998). Higher rates of violent deaths may have remained in chiefdoms (Gómez et al., 2016), and they only significantly declined when more complex states developed large-scale forms of capacity for organized violence.

Second, during and following this transition period there were changes in weaponry used. Projectile weapons including spears and stone tips, had been used already for ten-thousands of years. Bows and arrows, being in use thousands of years, were widely dispersed too, also because they were relatively easy to make. The use of these weapons did require skills, but they are surmised to have rather contributed to greater equality, as they enabled weaker men to attack a strong male from a distance (Scheidel, 2018, 27–8; Gat, 2008; and see Section 7). This changed with the military innovation during and after the rise of chiefdoms. These innovations either required more coordination, as the construction of defensive ditches from c. 6,000 BC, and later that of walls (Ferguson, 2013), or consisted of the introduction of new weaponry that because of its higher costs could more easily be monopolized, as the bronze weapons and, later, war horses and chariots. It is important to note, however, that the major introduction of bronze weaponry took place after the rise of chiefdoms. Chronologies differed per region, but the time gap mostly was some one or two millennia. Still, the monopolization of arms into the hands of violence specialists may have been sped up by the possibilities for restricting the use of this more costly (bronze) weaponry.

Third, disparities in material prosperity increased in chiefdoms. For the prehistorical and proto-historical period, these disparities can only be reconstructed by way of non-documentary, archaeological sources, which are not easy to interpret because of measurement issues (Fochesato, Bogaard, and Bowles, 2019). They do suggest, however, that inequality levels rose with the domestication of plants and animals and with increased socio-political scale (Kohler et al., 2017; Scheidel, 2018, 25–42; Bogaard et al., 2019). Inequality was substantially lower for small-scale societies of hunter-gatherers, foragers and horticulturalists. The latter category, however, where only modest wealth inequality and low intergenerational transmission of inequalities is found, shows that domestication in itself is not the cause of rising disparities (Gurven et al., 2010) and the same is suggested by the fact that in the first millennia after the Neolithic Revolution, as large chiefdoms and kingdoms had not yet emerged, wealth inequalities remained modest (Bowles and Choi, 2019). Rather, the crucial factor in rising inequalities is the combination of domestication and the later introduction of draft animals, the growing scale of societies and the emergence of hierarchies and the institutions that underpinned them, as most notably property rights that are upheld by the coalition of violence specialists.

Fourth, the archaeological record suggests that the prosperity of ordinary people did not rise with the Neolithic Revolution, the further intensification of agriculture, and the growth in scale of societal units in the following millennia. In contrast, the Neolithic Revolution and the subsequent waves of agricultural intensification raised land productivity but probably not labor productivity, and the required time investment to obtain sufficient food for most people became larger. Skeleton remains indicate declining biological living standards (Cohen and Crane-Kramer, 2007), even though a recent analysis points to stature increasing again in the Near East and Europe after the fourth millennium BC (Rosenstock et al., 2019). In interpreting these data, however, it is difficult to separate between the effects of agricultural intensification and rising population densities on the one hand and the effects of emerging larger political units on the other. More intensive work and the spread of diseases as a result of higher population densities fall into the first category. Increased security from open violence, heightened rates of extraction and rising inequalities fall into the second. Particularly as a result of the growing inequalities, the net effect of both processes combined was probably not favorable for the living standards of the majority of people, which helps understanding why tribes were not particularly fond of being incorporated in states and their societies kept on living side-by-side with these states for long.

The preceding historical sketch shows how production, surpluses, and extraction are vital components to understand the development of chiefdoms. This highlights the crucial importance of the appropriability of production, which can in its turn be related to crops that were introduced or gained in importance after the Neolithic Revolution, most particularly grain (Mayshar et al., 2022; Scott, 2017, 21–2, 128–36). Another exception is formed by those localities where rich resources were found within narrowly confined places which could easily be controlled and monopolized, thus enabling the rise of hierarchies, as happened with rich fishing sites along the Pacific coast of North America (Smith and Codding, 2021). The spatial concentration of these resources played a role similar to that of appropriable and storable grain. Grain production, including wheat, barley and rice, is spatially concentrated, its ripening is observable, and its harvesting happens in concentrated time periods. Consequently, grain is easy to tax. Moreover, because of its high value per volume it is easy to transport and store. Tubers, including crops like yam, potato, cassava and yucca, on the other hand, can be easily concealed dispersed underground, ripen over a lengthy period and can be left in the ground and remain edible for more than a year, making them hard to find and tax.

The reflection of this in the historical record is that hierarchical organization within large chiefdoms developed, often independently from each other, in almost all locations where the Neolithic revolution gave rise to the prevalence of grain cultivation. A partial exception is formed by Peru, where tubers, and particularly potatoes, formed a major crop, but quinoa and particularly maize played a similar role as appropriable crop as grain did elsewhere (Scott, 2017, 23, 128–33). The link between hierarchy and grain production is also confirmed, conversely, by the only real exception among the areas that saw a very early adoption of agriculture but had no evolution beyond tribal societies. This was New Guinea, and this is exactly the area which saw a Neolithic Revolution without a substantial role of grain production due to the lack of domesticable large-seeded plants. This crucial importance of the appropriability of the dominant crop is recently confirmed by Mayshar et al. (2022) who show that increased agricultural output was not associated with the emergence of hierarchical societies, but that grain cultivation was. Larger chiefdoms developed where grain cultivation developed.

The existence of a grain-hierarchy nexus is also apparent in the places where farmers concentrated on cattle herding instead of arable crops. The part of the globe where such a 'pastoral Neolithic' is most apparent is Eastern Africa, where the earliest food producers generally were cattle herders, living alongside groups of hunter-gatherers from the eight millennium BC on (Marshall and Hildebrand, 2002). Even though some of these pastoral societies saw wealth disparities, observed from the fourth millennium BC, there is no evidence for the development of social stratification, except for the Nile Valley, the African region where grain by exception was introduced early, and perhaps the few African regions where cattle herding was preceded by grain cultivation and became complementary to it (Marshall and Hildebrand, 2002). These studies thus do not contradict a general association between grain production and the rise of chiefdoms and hierarchy. Still—to repeat—grain cultivation did not necessarily or automatically lead to the emergence of chiefdoms and hierarchy, as also evidenced by the gap of several millennia between the introduction of grain cultivation and their emergence; something we aim to elucidate by way of our model.

3 The game

Our analysis of the emergence of a monopoly of violence in the hands of a dominant coalition uses an infinitely repeated game of individual investment and robbery with $N > 2$ *ex ante* identical and risk neutral players; $N > 2$ opens the possibility for the formation of coalitions, which will be critical in our analysis. While there is an over-arching game encompassing all equilibria that we study, the complete game is most easily explained by developing it in three steps. The first two models only use the stage game and do not require the repeated game structure. Model 1 has only investment in either edible resources or in arms used for robbery and the defense against robbery. Model 2 adds the option of investing in monitoring, which informs the investing player about the actions of the player(s) he chooses to monitor. The repeated-game structure is only used in Model 3, when reputations are indispensable to avoid violence specialists to abuse their violence capacity. Model 3 also introduces the possibility of 'voluntary' grain transfers and communication between players. Model 1 is developed in this section, the next two models are developed in the two subsequent sections. For each model, we discuss one typical equilibrium. While we do not claim an historical analogue for the Complete Anarchy Equilibrium analyzed for Model 1, the equilibria analyzed for Model 2 and 3, the Single Tribe Equilibrium and the Single Chiefdom Equilibrium respectively, are central to our historical argument. Finally, we refer to players as males. This is not a routine application of a linguistic convention, but a deliberate choice: violence has been a heavily male dominated activity.

3.1 Model 1: investment and robbery

Model 1 comprises of two consecutive phases, where players move simultaneously:

1. *Investment Phase*: players choose their investments.
2. *Robbery Phase*: players decide on whether to attack, and if so, who to attack.

After the Investment Phase, players do not observe the investment decisions of other players. After the Robbery Phase, players receive their payoff and the game ends. The expected payoff U_i comprises two consumables grain G_i^C and tubers B_i:

$$U_i := G_i^C + B_i - \frac{1}{2}\beta B_i^2, \tag{1}$$

where $\beta \in (0,1)$ is a parameter signifying the declining marginal payoff of tubers relative to grain.

During the *Investment Phase*, each player i has a unit endowment that can be invested in grain, tubers and arms, $G_i \geq 0, B_i \geq 0, A_i \geq 0$ respectively:

$$G_i + B_i + A_i \leq 1, \tag{2}$$

Equation (2) implies that G_i, B_i and A_i vary continuously on the unit interval. Hence, the action space of the game is infinite. Individual optimization implies that players will invest their full endowment; thus (2) holds with equality in equilibrium.

Although both G_i and B_i are produced from player's initial endowment, they have starkly different characteristics along two dimensions. First, tubers yield a lower payoff than grain and they have a declining marginal payoff: $\mathrm{d}U_i/\mathrm{d}B_i = 1 - \beta B_i$. Second, investment in grain is contestable, whereas that in tubers is not. Hence, investment in tubers B_i feeds directly into i's payoff. Instead, his payoff from grain derives from actually consumed grain G_i^C, which might differ from his initial investment G_i due to robbery, either from him by other players or by him from others.

During the *Robbery Phase*, i decides how to use his investment in arms A_i. He can use his arms either to attack one other player j or to defend himself or one other player. If i's attack is successful, it yields him a share of j's investment in grain G_j as a booty. Formally, let $I_{ij} \in \{-1, 0, 1\}$ be a ternary variable taking the value 1 if i plans to attack j, -1 if i plans to defend j and 0 otherwise. Since i must pick one player to attack or defend, so that $\Sigma_{jt} I_{ij}^2 = 1$; he may choose to self-defend, $I_{ii} = -1$, but cannot choose to attack himself, $I_{ii} \neq 1$.

The exact rules of engagement are detailed in Section 3.1.1. For the moment, let $R_{ij} \geq 0$ be the booty taken from i by j. The sum of the booties taken from i cannot exceed his grain budget: $\Sigma_j R_{ij} \leq G_i$. Accordingly, i's grain consumption after robbery reads

$$G_i^C := G_i + \Sigma_j \left(R_{ji} - R_{ij}\right). \tag{3}$$

Since $R_{ji} \geq 0$ and $\Sigma_j R_{ij} \leq G_i$, $G_i^C \geq 0$. Equation (3) implies that grain robbed from i is given one-for-one to j. Robbery itself is therefore purely redistributive—it does not cause a Hicks-Kaldor efficiency loss. Only the investment in arms is costly, since part of the endowment is spent on arms rather than on edible resources. The act of attacking itself is free.

3.1.1 The rules of engagement

Here, we formulate the exact mapping of investments in arms and patterns of attack and defense between all N players into the booties R_{ij}. This mapping is a daunting task, since the number of possible actions a single player can take in the Robbery Phase is $2N-1$.[3] Hence, the number of possible patterns for N players is $(2N-1)^N$. Even for $N = 3$, this is 125. The analysis of these patterns become feasible by a careful specification of the rules of engagement.

A crucial rule is the *Priority of Self-defense:* player i can only realize an attack or defense on j if i can successfully defend against attacks by other players on i himself. This rule is part of the rules of the game that constrain i's action set, it is not the consequence of i's strategic choices. It rules out strategies where i can ignore attacks on himself—for example because he has not invested in grain, and therefore does not care to be robbed—while simultaneously robbing grain from other players. This rule allows a group of players to defend against i who randomly attacks one member of the group. Without the Priority of Self-defense, each member would have

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[3] A player can choose one of $N-1$ players to either attack of defend, yielding $2(N-1)$ options, or choose to defend herself, yielding one additional option.

to defend individually against i's potential attack. With this rule, in contrast, the group can attack i to neutralize his potential attack—even more, it suffices that one member of the group attacks i with sufficient arms.

The Priority of Self-defense is implemented by differentiating between *planned* and *executed* attacks and defenses. Player i's *planned* attack or defense is equal to his investment in arms A_i. Let Y_i be i's *executed* attack or defense and let X_i be the sum of the executed attacks on i by other players. Y_i and X_i satisfy:

$$\begin{aligned} Y_i &:= \quad \max\left[A_i - X_i, 0\right], \\ X_i &:= \quad \Sigma_j \mathcal{I}\left[I_{ji} = 1\right] Y_j, \end{aligned} \tag{4}$$

where $\mathcal{I}[x]$ is an indicator function, taking the value 1 if x is true and 0 otherwise. Player i's executed attack or defense Y_i is therefore equal to his investment in arms A_i minus the total executed attack X_i of all other players on i.[4] Since a planned attack or defense can never be negative, Y_i is bounded below at zero. The total executed attack on i is the sum of the executed attacks on i by all other players. Equation (4) describes a system of $2N$ equations with Y_i and X_i for all i as unknowns.

Proposition 1: planned versus executed attacks/defenses
There exists a unique solution to equation (4) for X_i and Y_i.

Proof: see Appendix A.1.

We give a rough outline of the proof. We start from the players that are not attacked by any other player. By construction, their executed attack is equal to their planned attack. As a second step, we then adjust the attack strength of the players attacked by this first group by subtracting the executed attack of the first group from the arms investment of the players that they attack. We can repeat this procedure for the players in this second group who are only attacked by players from the first group. Their executed attack is their planned attack minus the attack strength from players in the first group. We repeat this procedure until we are left with a group wherein each player is attacked by one other player from this group. This group can be subdivided in one or more circular patterns, where i attacks j, j attacks k, and k attacks i. We show that also for these circular patterns, there exists a unique solution.

Given the solution of (4), the step from Y_j to the booties R_{ij} is simple. Let S_i be the strength of the *net executed attack* on i:

$$S_i := \Sigma_{j,j \neq i} I_{ji} Y_j - A_i, \tag{5}$$

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[4] Note that i's capacity to execute an attack or defense is not affected by other players' defense of i. Their defence helps to protect i's investment grain G_i, but not to maintain Y_i.

S_i differs from X_i by subtracting both the defenses of i by other j (since $I_{ji} = -1$ codes a defense) and i's self-defense, compare (4). When j plans to attack i ($I_{ji} = 1$), his booty R_{ij} satisfies

$$R_{ij} := \mathcal{T}\left[\gamma S_i\right] \frac{Y_j}{X_i} G_i. \tag{6}$$

where $\gamma > 1$ is a parameter. $\mathcal{T}[x]$ is a kinked S-shaped function of x, which takes the value 0 if $x < 0$, x if $0 \leq x \leq 1$ and 1 if $x > 1$. The kinked S-shape of $\mathcal{T}[\gamma S_i]$ plays a critical role in our analysis.

The right hand side of equation (6) is made up of three factors:

1. G_i is the investment in grain that is exposed to robbery;
2. $\mathcal{T}[\gamma S_i]$ is the fraction of i's grain that is robbed by other players. The kinks in $\mathcal{T}[\gamma S_i]$ at $\gamma S_i = 0$ and $\gamma S_i = 1$ account for the fact that the booty can neither be smaller than zero nor larger than G_i;
3. since $Y_j \geq 0$ and $\Sigma_j \mathrm{I}\left[I_{ji} = 1\right] Y_j / X_i = 1$ by (4), Y_j/X_i is j's share in the booty. When there are multiple attackers, the booty is divided proportionally to their executed attack Y_j.

These rules yield three important implications:

1. equation (6) implies $0 \leq \Sigma_j R_{ij} \leq G_i$;
2. the Priority of Self-defense rules out that i simultaneously robs and is robbed himself, since if $Y_i > 0$ then $S_i < 0$ and if $S_i > 0$ then $Y_i = 0$, see (4) and (5);
3. equation (6) implies that if i invests his full endowment in grain $G_i = 1$ and no other players attack i, than an investments in arms by j of $A_j = \gamma^{-1} < 1$ to attack i is profitable, since the cost of this investment is lower than its reward: $A_j < G_i$. This is a critical feature of our model.

Using this last implication, we can analyze two benchmark outcomes of this game for future reference. In the first, First Best FB, players' full endowment is invested in grain; in the second, the Outside Option OO, it is invested in tubers:

$$G_{\mathrm{FB}} = U_{\mathrm{FB}} = 1, \tag{7}$$

$$B_{\mathrm{OO}} = 1, \qquad U_{\mathrm{OO}} = 1 - \frac{1}{2}\beta. \tag{8}$$

The latter is called the Outside Option since the investment in tubers offers protection against robbery. For any equilibrium, the equilibrium payoff of each player must be at least equal to U_{OO}, since a player always has the option to invest his full endowment in tubers. This offers a powerful break on the extractive capacities of other players. Both full investment in grain and in tubers, however, are not Nash equilibria. When all players invest fully in grain, a deviant who invests in arms and robs another player gets a higher payoff. Similarly, when all players invest fully in tubers, a deviant investing his full endowment in grain gets a higher payoff, as nobody invests in arms to rob his grain.

3.1.2 The first order conditions for optimal investment

Player i has to decide on his investments and attacking plans knowing neither the investments nor the attacking plans of other players. Accordingly, he has to form Bayesian beliefs about these investments and plans in order to decide on his optimal response. We therefore have to derive the complete probability distribution of investments and attacking plans of all other players. In general, this is an unrealistic objective. Luckily, the equilibria discussed in this paper, both in Model 1 as well as in the two subsequent models discussed in the next two sections, satisfy a feature that greatly simplifies this task.

The Characteristic of Known Attack Strengths and Coalition Sizes
When i decides on the investment of his initial endowment and when he plans to rob j, $I_{ij} = 1$, he does not know whether this robbery will succeed. However, if his robbery of j succeeds, he knows with certainty: (i) the size K of the robbing coalition that joins him in his attack, (ii) the investment in arms A_k of the $K-1$ other members of this coalition, and (iii) the investment in arms A_j that j has available for defense. Similarly, i does not know whether or not he will be robbed. However, if he is robbed, he knows with certainty what will be the net force S_i of this robbery.

The Characteristic of Known Attack Strengths and Coalition Sizes implies the Bayesian beliefs are fully characterized by just two statistics:

1. the probability P_i that i will be robbed successfully;

2. the probability P_{ij} that i's planned robbery of j will succeed.

Given the second implication of the rules of engagement that i cannot simultaneously rob himself and being robbed, $\mathrm{P}_i + \mathrm{P}_{ij} \leq 1$, where $1 - \mathrm{P}_i - \mathrm{P}_{ij}$ is the probability that i neither robs himself nor is being robbed.

The *Characteristic of Known Attack Strengths and Coalition Sizes* greatly simplifies the analysis. It is neither imposed by the rules of the game, nor a condition for an equilibrium. It just happens to be implied by the strategies of all players in the equilibria that we study.

Using this structure of the Bayesian beliefs, we can derive the marginal expected payoffs of G_i, B_i and A_i. For $I_{ij} = 1$, U_i satisfies[5]

$$\begin{aligned} U_i &= \left(1 - \mathrm{P}_i \mathcal{T}\left[\gamma S_i\right]\right) G_i + \mathrm{P}_{ij} \mathcal{T}\left[\gamma S_j\right] \frac{A_i}{X_j} G_j + B_i - \frac{1}{2}\beta B_i^2, \\ S_i &= K A_k - A_i, \quad S_j = (K-1) A_k + A_i - A_j, \quad X_j = (K-1) A_k + A_i. \end{aligned} \tag{9}$$

The expected marginal return on A_i is the derivative of U_i with respect to A_i:

$$\begin{aligned} \frac{\mathrm{d}U_i}{\mathrm{d}A_i} &= \gamma \mathrm{P}_i \mathcal{M}\left[\gamma S_i\right] G_i + \mathrm{P}_{ij} \left(\gamma \mathcal{M}\left[\gamma S_j\right] \frac{A_i}{X_j} + (K-1) \mathcal{T}\left[\gamma S_j\right] \frac{A_k}{X_j^2} \right) G_j, \\ \mathcal{M}[x] &:= \mathrm{d}\mathcal{T}[x] / \mathrm{d}x. \end{aligned} \tag{10}$$

Hence, $\mathcal{M}[x] = 1$ for $0 < x < 1$ and $\mathcal{M}[x] = 0$ for $x < 0$ and $x > 1$; $\mathcal{M}[x]$ is discontinuous at $x = 0$ and $x = 1$.[6] These discontinuities correspond to the kinks in $\mathcal{T}[x]$. The first term of $\mathrm{d}U_i / \mathrm{d}A_i$ is i's return on defending when being robbed and is:

1. zero for $\gamma S_i < 0$ since $\mathrm{P}_i = \mathcal{M}\left[\gamma S_i\right] = 0$: the robbery fails anyway;
2. proportional to $P_i G_i$ for $0 < \gamma S_i < 1$ since $\mathrm{P}_i = \mathcal{M}\left[\gamma S_i\right] = 1$: investment in arms reduces the booty;
3. zero for $\gamma S_i > 1$ since $\mathcal{M}\left[\gamma S_i\right] = 0$: robbers take the full booty anyway.

The second term is i's return when robbing j. Again, it is made up of three parts:

1. zero for $\gamma S_j < 0$ since $P_{ij} = 0$: the robbery fails;
2. high for $0 < \gamma S_j < 1$: A_i increases both the booty and i's share A_i / X_j in it;
3. low for $\gamma S_j > 1$ since $\mathcal{M}\left[\gamma S_j\right] = 0$: A_i does not increase the booty, but only i's share A_i / X_j.

On the trajectory $0 < \gamma S_j < 1$ the marginal revenue $\mathrm{d}U_i / \mathrm{d}A_i$ is increasing in A_i, since a higher A_i increases both the total booty as i's share A_i / X_j in it. The second order condition for an optimum is likely not to be satisfied there. On the trajectory $\gamma S_j > 1$, to the contrary, the marginal revenue $\mathrm{d}U_i / \mathrm{d}A_i$ is decreasing in A_i, since $\mathcal{M}\left[\gamma S_j\right] = 1$ and because $\mathrm{d}\left(A_i / X_j\right) / \mathrm{d}A_i$ is decreasing in A_i: the larger i's share

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[5] The first line starts from (1). The first two term measures G_i^C. They apply (3): $\Sigma_k R_{ik} = \mathrm{Pr}_i \mathcal{T}\left[\gamma\left(X_i - A_i\right)\right] G_i$ and $R_{ji} = \mathrm{Pr}_{ij} \mathcal{T}\left[\gamma\left(X_j - A_j\right)\right] \frac{X_i}{Y_j} G_j$, see (6).

[6] We use:

$$\begin{aligned} \mathrm{d}\mathcal{T}\left[\gamma S_i\right] / \mathrm{d}A_i &= \mathrm{d}\mathcal{T}\left[\gamma\left(K A_k - A_i\right)\right] / \mathrm{d}A_i = -\gamma \mathcal{M}\left[\gamma S_i\right], \\ \mathrm{d}\mathcal{T}\left[\gamma S_j\right] / \mathrm{d}A_i &= \mathrm{d}\mathcal{T}\left[\gamma\left((K-1) A_k + A_i - A_j\right)\right] / \mathrm{d}A_i = \gamma \mathcal{M}\left[\gamma S_j\right]. \end{aligned}$$

A_i/X_j in the booty, the more costly it is to increase it even further. Here, the second order condition is satisfied.

Since the marginal costs of G_i, B_i and A_i are identical, see (2), the optimal investment sets equal their expected marginal payoffs. For G_i and A_i we obtain:

$$1 - \mathrm{P}_i \mathcal{T}\left[\gamma S_i\right] = \mathcal{T}\left[\mathrm{d}U_i/\mathrm{d}A_i\right], \tag{11}$$

The left hand is the marginal expected payoff of G_i, see (9). For G_i and B_i, we obtain:

$$1 - \mathrm{P}_i \mathcal{T}\left[\gamma S_i\right] \leq 1 - \beta B_i \Rightarrow B_i = \mathcal{T}\left[\beta^{-1}\mathrm{P}_i \mathcal{T}\left[\gamma S_i\right]\right], \tag{12}$$

Again, the left and the right hand are the marginal expected payoffs of G_i and B_i respectively. The larger the probability of being robbed and the larger the share of the grain that is robbed, the more i invests in tubers. Equation (11) and (12) are the backbone of the analysis of many equilibria, both in this and in subsequent sections.

3.2 The Complete Anarchy Equilibrium CA-E

We discuss one equilibrium for Model 1, where each player randomly attacks another player. Since all players invest equally in arms, their only chance of a succesful attack is by accidentally teaming up with another player, using their combined strength to overwhelm their victim. This equilibrium might be interpreted as the classical Hobbesian state of nature. Though we do not claim any analogy for this equilibrium in human history, the equilibrium is useful to analyze the critical role of coalitions in the use of violence and the type of institutional problems that early human societies had to resolve to contain robbery.

The discontinuity in the composition of marginal revenue at $x = 1$ plays a critical role in this equilibrium. Consider (10). All players make the same investment in grain —$G_i = G_j = G_{\mathrm{CA}}$— while the size of an attacking coalition is $K = 2$. There are two possible patterns of attacks: either a *circular attack*—see the discussion of the proof of Proposition 1—or one player being attacked by two other players. Due to players' random choice who to attack, a robbery occurs with probability $\frac{3}{4}$;[7] in that case, i has probability $\frac{2}{3}$ of being one of the two robbers and probability of $\frac{1}{3}$ of being robbed. Hence, $\mathrm{P}_{ij} = \frac{3}{4}\frac{2}{3} = \frac{1}{2}$, $\mathrm{P}_i = \frac{3}{4}\frac{1}{3} = \frac{1}{4}$. Suppose $A_k = A_j = A_i = A_{\mathrm{CA}} = \gamma^{-1}$. Hence, just to the left and the right respectively of the discontinuity at $A_i = \gamma^{-1}$,

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[7] Consider player i (the other players are symmetric). Consider the case $I_{ij} = 1$ (the case $I_{ik} = 1$ is symmetric); both j and k can attack either of two other players, yielding $2^2 = 4$ combinations. Only one of these combinations is a circular attack, the other three yield a combined attack of two players on the third.

(10) simplifies to:[8]

$$\frac{\mathrm{d}U_i}{\mathrm{d}A_i} = \begin{cases} \lim A_i \uparrow \gamma^{-1} & : \quad \frac{3}{4}\gamma \mathrm{P}_{ij} G_{\mathrm{CA}} \quad = \quad \frac{3}{8}\gamma G_{\mathrm{CA}} \\ \lim A_i \downarrow \gamma^{-1} & : \quad \gamma\left(\mathrm{P}_i + \frac{1}{4}\mathrm{P}_{ij}\right) G_{\mathrm{CA}} \quad = \quad \frac{3}{8}\gamma G_{\mathrm{CA}} \end{cases}. \tag{13}$$

The discontinuity in the composition does therefore not translate into a discontinuity in its level. The fall in the return of attacking j at $A_i = \gamma$ is exactly offset by the increase in the return of i defending himself. Since the marginal revenue $\mathrm{d}U_i/\mathrm{d}A_i$ is increasing at the trajectory $0 < \gamma S_j < 1$, see the discussion of (10), one can expect $\gamma S_j > 1$ in equilibrium, implying $A_{\mathrm{CA}} > \gamma^{-1}$. The subsequent proposition characterizes this equilibrium.

Proposition 2: the Complete Anarchy Equilibrium CA-E
Let $N = 3$ and let

$$\beta > 1/4, \qquad \gamma < \frac{32\beta}{4\beta - 1}. \tag{14}$$

The following strategy played by all three players is mixed strategy Nash-E:

Investment Phase:

$$A_{\mathrm{CA}} = 1 - (4\beta)^{-1} - 2\gamma^{-1}, \qquad B_{\mathrm{CA}} = (4\beta)^{-1}; \tag{15}$$

Robbery Phase: i randomly selects another player $j \neq i$ and sets $I_{ij} = 1$.

The payoff reads

$$U_{\mathrm{CA}} = \frac{7}{32}\beta^{-1} + 2\gamma^{-1}. \tag{16}$$

Here, and in all propositions on subsequent equilibria we shall not explicitly state the value of G since it can be solved as a residual item from the budget constraint (2).

Proof:
The equilibrium is proven by backward induction:

Robbery Phase: due to the Priority of Self-defense, planning to attack another player is always optimal since it offers i an opportunity for an additional payoff by robbing j since i's arms will automatically redirected to self-defense when needed.

Investment Phase: since all players invest equally in arms and grain, i's planned attack on j can only succeed if the third player happens to join him in his attack,

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[8] Though circular attacks are unsuccessful, one might conjecture that there still is an effect on players' incentives to invest in arms: by investing more, a player can rob from other players who invest less in case of a circular attack. Appendix A.1 shows that this intuition is incorrect for $N = 3$. If i invests more in arms to attack j, j will have to divert his arms to self-defense. Hence, his attack on k is reduced, so that k's attack on i increases. Hence, i has to divert part of his arms to self-defense. This mechanism provides a defensive buffer up $A_i > A_j + A_k$.

see the discussion of (13) and the accompanying derivation of P_i and P_{ij}. Equation (15) and (14) imply $A_{\mathrm{CA}} > \gamma^{-1}$ and thus $\mathcal{T}[\gamma S_i] = \mathcal{T}[\gamma A_{\mathrm{CA}}] = 1$. Hence, by (12) $B_{\mathrm{CA}} = (4\beta)^{-1}$. By (11), and (13):

$$\frac{3}{4} = \frac{3}{8}\gamma G_{\mathrm{CA}} = \frac{3}{8}\gamma\left(1 - (4\beta)^{-1} - A_{\mathrm{CA}}\right).$$

substituting G_{CA} for (2) and B_{CA} for (15) in the second equality. Some rearrangement yields (15). Condition (14) implies $A_{\mathrm{CA}} > \gamma^{-1}$, as is required for the second order condition.

Since robbery is purely redistributive, $U_{\mathrm{CA}} = 1 - A_{\mathrm{CA}} - \frac{1}{2}\beta B_{\mathrm{CA}}^2$. Substitution of (15) and some rearrangement yields (16). U_{CA} must be higher than the payoff of a deviant who does not invests in arms at all—and thus sets $B_d = (4\beta)^{-1}$:

$$U_d = (1 - B_d)\mathrm{P}_i + B_d - \frac{1}{2}\beta B_d^2 = \frac{1}{4} - \frac{1}{32\beta},$$

which must be smaller than U_{CA}. Using (16), we obtain:

$$\frac{1}{4} < \frac{1}{16\beta} + \frac{2}{\gamma} \quad \Rightarrow \quad \gamma < \frac{32\beta}{4\beta - 1},$$

which is always true given and (14). ■

The extension of the analysis to higher values of N is conceptually straightforward, but a combinatorial nightmare—for example, we loose the *Characteristic of Known Attack Strengths and Coalition Sizes*, since for $N = 4$, a player can be attacked by coalitions of either 2 or 3 other players.

4 Model 2: adding monitoring

Model 2 introduces endogenous monitoring. During the Investment Phase, i can also invest in monitoring the actions of other players at a cost of $\frac{\mu}{N-1}$ where $\mu \in (0,1)$ is a parameter. Hence, player i's budget constraint reads

$$G_i + B_i + A_i + \mu\frac{M_i}{N-1} = 1, \tag{17}$$

where M_i is the number of other players i chooses to monitor. The parameter μ measures the cost of monitoring all other players as a share of the initial endowment. Since we have assumed $\mu \in (0,1)$, i can afford to monitor all other players by setting $M_i = N - 1$. Monitoring does not enter the payoff function of a player. Accordingly, each player maximizes the payoff (1), as in Model 1, subject to the updated budget constraint (17).

Let $M_{ij} \in \{0,1\}$ be a binary variable, taking the value one if i decides to monitor j and taking the value zero otherwise. Hence: $M_i = \Sigma_j M_{ij}$. If $M_{ij} = 1$, then i observes j's investment at the end of the Investment Phase. Hence, i can condition his actions in the Robbery Phase on j's actions in the Investment Phase. We assume j knows whether or not i monitors him. The merit of this assumption is that it allows i to commit to his claim that he is monitoring j since j can verify this claim at zero cost.

We parameterized the monitoring cost μ as the cost of monitoring all other players. The more natural approach would be to introduce a parameter $\chi := \frac{\mu}{N-1}$ for the monitoring-cost per person. Then, χ^{-1} is the maximum span of control of a single player, since (17) implies $M_i \leq \chi^{-1}$. We allude to this interpretation in our discussion. However, using μ rather than χ comes with the advantage that we take the cost of monitoring of just a few other players to be small. In one equilibrium discussed in Section 6, we ignore terms of $\mathcal{O}\left(N^{-1}\right)$. There, using $\frac{\mu}{N-1}$ rather than χ allows us to ignore the monitoring cost of just a single player.

Note that with this extension, CA-E still exists. Since all players invest the same amounts in tubers and arms, the Bayesian beliefs about these investments coincide with their realizations. Monitoring these investment does not add new information to the investing player. Hence, players do not invest in monitoring: $M_{\mathrm{CA}} = 0$.

4.1 The Single Tribe Equilibrium ST-E

The equilibrium that we discuss for Model 2 is the Single Tribe Equilibrium ST-E. In fact, this is one of two pivotal equilibria in the paper, as it describes the social structure of tribal societies. Our review of the empirical evidence in Section 2 suggests that contrary to the CA-E analyzed in the previous section, these societies contained violence by collective action against deviants. The ST-E shows how this can be achieved. It requires that all tribe members monitor each other so that deviations can be punished by coordinated collective actions.

Proposition 3: the Single Tribe Equilibrium ST-E
Let

$$\gamma > \frac{N}{N - \mu(N-1)}, \qquad \frac{1-\gamma^{-1}}{N-1} + \mu < \frac{1}{2}\beta. \tag{18}$$

The following strategy played by all i is a Bayesian Perfect Equilibrium BP-E:

Investment Phase:

$$A_{\mathrm{ST}} = \left(1-\gamma^{-1}\right) / \left(N-1\right), \qquad B_{\mathrm{ST}} = 0, \qquad M_{\mathrm{ST}} = N-1; \tag{19}$$

Robbery Phase: Along the equilibrium path, i defends himself: $I_{ii} = -1$. If another player d has deviated from the equilibrium investments in the Investment Phase, i plans to attack that player: $I_{id} = 1$.

Out-of-equilibrium beliefs: players belief that all other players attack a deviant d.

The payoff reads

$$U_{\mathrm{ST}} = 1 - \left(1 - \gamma^{-1}\right) / (N-1) - \mu. \tag{20}$$

Proof: the equilibrium can proven by backward induction.

Investment Phase: Since along the equilibrium path all players have invested the same amount A_{ST} in arms in the Investment Phase, a planned attack on another player fails. This action is therefore weakly dominated by self-defense. If a player d deviates on either A_{ST} or M_{ST} in the Investment Phase, player i observes this because he monitors all players $M_{id} = 1$. Since attacking is free, planning to attack d weakly dominates self-defense.

Robbery Phase: There is no robbery on the equilibrium path. Therefore, by equations (1) and (17), the optimal investment in tubers is zero. This yields an equilibrium payoff:

$$U_{\mathrm{ST}} = 1 - A_{\mathrm{ST}} - \mu.$$

Since any deviation on M_{ST} and/or A_{ST} invokes an attack by all other players in the Robbery Phase anyway, the most attractive deviation for d is not to monitor at all ($M_d = 0$)—since this yields the highest cost saving—and choosing the optimal amount A_d and planning to attack a random j in the Robbery Phase. Since all other players will attack d, the net attack strength on j satisfies $S_j = A_d - (N-1)\,A_{\mathrm{ST}}$. Hence, d sets A_d to maximize U_d

$$U_d = 1 - A_d + U_{\mathrm{ST}}\mathcal{T}\left[\gamma S_j\right],$$

since $G_{\mathrm{ST}} = U_{\mathrm{ST}}$ and $G_d = 1 - A_d$; $U_{\mathrm{ST}}\mathcal{T}\left[\gamma X_j\right]$ is the grain robbed by d from j.

Conjecture: $\gamma U_{\mathrm{ST}} > 1$. Then, the marginal cost of arms is less than d's marginal revenue up till the point where d robs all j's grain G_{ST}. Hence:

$$\begin{aligned} 1 &= \gamma\left[A_d - (N-1)\,A_{\mathrm{ST}}\right], \qquad (21)\\ U_d &= 1 - A_d + U_{\mathrm{ST}}. \end{aligned}$$

A Bayesian Perfect equilibrium requires that this deviation yields a weakly lower payoff: $U_d \leq U_{\mathrm{ST}}$. An efficient choice of A_{ST} makes this inequality binding, implying $A_d = 1$. Plugging this back into (21), solving for A_{ST}, and plugging this back into

(20) yields (19). We conjectured $\gamma U_{\mathrm{ST}} > 1$ and U_{ST} must be weakly higher than U_{OO}. Both are satisfied by condition (18).■

By monitoring all other players and by investing a small bit of their endowment in arms, players can collectively defend against an attack of a deviant who refuses to monitor and deviates from the norm on investment in arms. While there was permanent violence and robbery in the more or less hypothetical Hobbesian CA-E, forcing players to invest part of their endowment in tubers, players invest their full endowment that is left after monitoring and arms in the ST-E. In that sense, the institution of monitoring is a prerequisite for a full scale Neolithic revolution.

For small monitoring-cost μ, the critical deviation in the CT-E is investing more in arms to rob another player, as in Proposition 3. For larger μ, the critical deviation is investing less in arms and free-riding on other player's deterrence of deviants. Deterrence is therefore a public good, in particular for larger μ, see (18).

Proposition 3 specifies the minimum value of A_{ST} needed for a the single tribe equilibrium. A tribe can also operate a higher norm for A_{ST}, which also constitutes an equilibrium because no tribesman can afford to deviate. However, this norm would be less efficient as it would require an additional wasteful investment in arms. One would expect a tribe to coordinate on the efficient norm.

For future reference, we observe that A_{ST} is of order $\mathcal{O}\left(N^{-1}\right)$: a small investment in arms by many players is enough, since the coordinated attack of N players on a single deviant is a sufficient deterrent. By taking the cost of monitoring per person $\chi = \frac{\mu}{N-1}$ rather than μ as a constant, an increase in N also increases the cost of monitoring. Hence, the dependence on N is also the weak spot of ST-E. For larger N, the cost of monitoring gets out of hand, while the cost of arms becomes irrelevant. The payoff U_{ST} is therefore hump-shaped in N. At a low number of players, increasing N reduces A_{ST} and thus increases U_{ST}. As N grows larger, the increase in monitoring-cost reduces U_{ST}, until it gets below U_{OO}. At that point, the tribal society collapses.

5 Model 3: adding transfers and repeated game

5.1 Transfers and messages

Model 3 introduces three new elements to the game. First, we introduce two new phases in between the Investment and the Robbery Phase—the *Taxation Phase* and the *Wage Payment Phase*—during which players can transfer grain to other players. This extension allows for asymmetric and hierarchical behavior within the stage-game. Second, we introduce the option of players to send each other messages. Finally, we introduce a repeated game structure. This subsection discusses the first two extensions. The next subsection discusses the repeated game structure.

The *Taxation Phase* and *Wage payment Phase* are two consecutive, but otherwise similar, phases during which i can decide to pay taxes $T_{ij} \geq 0$ or wages $W_{ij} \geq 0$ to one or more other player(s) j. The sum of i's tax payments T_{ij} can never exceed his investment in grain: $\Sigma_j T_{ij} \leq G_i$, while the sum of his wage payments W_{ij} can never exceed his grain endowment after accounting for tax transfers: $\Sigma_j W_{ij} \leq G_i^T$, where G_i^T is defined as:

$$\begin{aligned} G_i^T &: = G_i + \Sigma_j (T_{ji} - T_{ij}), \qquad (22) \\ G_i^W &: = G_i^T + \Sigma_j (W_{ji} - W_{ij}), \\ G_i^C &: = G_i^W + \Sigma_j (R_{ji} - R_{ij}). \end{aligned}$$

Since $T_{ji} \geq 0$ and $\Sigma_j T_{ij} \leq G_i$, $G_i^T \geq 0$. Similarly, since $W_{ji} \geq 0$ and $\Sigma_j W_{ij} \leq G_i^T$, $G_i^W \geq 0$. Note that (22) is analogous to (3) for Model 1, only replacing i's investment in grain G_i by his grain endowment accounting for tax and wage transfers. All robbery therefore regards G_i^W rather than G_i; accordingly, G_i is replaced by G_i^W in equation (6)-(10).

Since the Taxation and Wage Payment Phase are exactly symmetric, one can wonder why we introduce both phases separately in our model. Why can they not be combined in single phase? The reason is that these transfers are constrained by non-negativity constraints—$\Sigma_j T_{ij} \leq G_i$ and $\Sigma_j W_{ij} \leq G_i^T$—which might be binding. As we shall see, the wage payments paid by the chief to a group of soldiers in the *Wage Payment Phase* are only feasible because the chief receives substantial tax revenues during the *Taxation Phase*. This interpretation of these transfers motivates referring to these phases as 'taxes' and 'wages', though from the point of view of the model, they could just as well be dubbed Transfer I and II respectively. We use the modern word 'taxation', but we could just as well have referred to them as 'bribes', 'tributes', 'gifts' or any other word used for this type of transfers throughout history.

Both tax and wage transfers are non-enforceable by receiving players: players can always decide not to pay. If j refuses to pay i, i can rob j in the Robbery Phase, but he cannot otherwise force j to pay. Player i observes that a transfer is paid to him, but not who makes this transfer, unless he monitors that player: $M_{ij} = 1$.

Players can send messages to other players at any time during the game. These messages are non-binding: neither the sender nor the receiver is obliged to change his actions in response to the message. Sending a message is free. The ability to send messages is critical, as it allows the chief to send orders to his soldiers, which they then may or may not follow up. Without this option, the chief would be unable to turn his information advantage into a higher payoff.

5.2 The repeated game and social roles

The previous subsection has completed the discussion of the stage game. In Model 3, this stage game is played repeatedly in infinitely many rounds. Let U_{it} and V_{it} be i's expected payoff during the current round and the expected lifetime payoff accumulated across the current round and all future rounds respectively, both at the start of round t. Future payoffs are discounted at a rate $\rho \in (0, 1)$ per round. Hence, i's expected lifetime payoff is equal to the expected payoff during the current stage plus the discounted value of the expected lifetime payoff at the start of next round:

$$V_{it} = U_{it} + (1 - \rho)\, V_{i,t+1}. \tag{23}$$

Players seek to maximize their expected lifetime payoff.

We are interested in *Bayesian Perfect equilibria* (BP-E) to this game. Player i's strategy specifies his actions in each phase, conditional on (i) the information available to him at that point in time, (ii) his Bayesian beliefs about variables on which he is uninformed, and (iii) the strategies played by other players. In any case, i is informed about grain transfers that he receives and grain that is robbed from him, but not about who makes these transfers and who robs him. Moreover, when i decides to monitor j at t—$M_{ijt} = 1$—he observes all actions taken by j at the end of each phase. However, in order to observe j's action at $t + 1$, i has to reinvest in monitoring—$M_{ij,t+1} = 1$. However, i can memorize information collected in previous rounds.

There are many equilibria of the BP-E type. We have no hope of being able to characterize them all. We restrict our attention to a subset of relatively simple BP-Es. First, we restrict ourselves to *stationary* BP-Es where all players take the same action every round *along the equilibrium path* of both the main game and all subgames. Hence $U_{it} = U_{is}$ and $V_{it} = V_{is}$ for all i and $s > t$ along the equilibrium path of each subgame. Then, (26) implies the expected lifetime payoffs along the equilibrium path of a stationary BP-E to be:

$$V_{it} = U_{it}/\rho. \tag{24}$$

Hence, we can eliminate V_{it} and focus on U_{it}.

Second, *stationary* BP-Es can be classified in two sub-classes. The first class are *Markov Perfect equilibria* (MP-E), where i's actions at t do not depend on his information on the actual actions taken by other players in previous rounds $s < t$. When all other players' strategies do not condition their actions on past actions of other players, it does not make sense for i to do so, since conditional on the strategies of other players i's strategy was the best response, irrespective of the actual behaviour of others in previous rounds. Hence, each round is a BP-E of its own. Since a single round is not an infinitely repeated game, we can apply backward induction to prove that a set of strategies is BP-E. In fact, the CA-E analyzed in Section 3 and the ST-E analyzed in Section 4 are MP-Es of Model 3.

The second class of equilibria are *Non-Markovian Equilibria* (NM-E), where i's strategy in round t depends on his information on other player's actions in previous rounds $s < t$. This may allow players to support more cooperative/efficient BP-Es then can be supported in a MP-E, since deviations can be punished in future rounds. Within this class, we restrict our attention to equilibria where i' strategy depends on the information on other players actions at $s = t - 1$ only, not on information about actions in earlier rounds $s < t - 1$.

A punishment shifts strategies from one BP-E to another. We make a further distinction of NM-Es in two sub-classes: *Symmetric* (S-NM-E) versus *A-Symmetric* (AS-NM-E). Both classes exploit information about actions taken in the previous round to sustain a more cooperative equilibrium, but in a very different way. In a symmetric S-NM-E, all players play the same strategy. Hence, their payoffs are the same: $U_{it} = U_{\mathrm{S-NM}}$ for all i and t. Suppose there exists:

1. an MP-E with a lower expected payoff $U_{\mathrm{MP}} < U_{\mathrm{S-NM}}$;

2. an action that yields a deviant d a higher payoff $U_{dt} > U_{\mathrm{S-NM}}$ at t.

Then, an S-NM strategy demands all players to switch to the MP strategy when just one player deviates from the S-NM strategy. As long as

$$\begin{aligned} V_{\mathrm{S-NM}} &\geq U_d + (1-\rho)\, V_{\mathrm{MP}} \Rightarrow \\ \rho\,(U_d - U_{\mathrm{S-NM}}) &\leq (1-\rho)\,(U_{\mathrm{S-NM}} - U_{\mathrm{MP}})\,, \end{aligned} \tag{25}$$

and using (23) in the second line, the threat of a transition to the MP-E is sufficient to prevent deviations. Since the punishment strategy is MP-E, the punishment is a credible threat: conditional on the strategy of other players, participating in the punishment is i's best response. Note furthermore that (25) implies that S-NM-E is BP-E because MP-E is BP-E. Condition (25) states that the short run gain of a deviation times the weight ρ of the current round must be less than the long run loss of a breakdown of the cooperation times the weight $1 - \rho$ of future rounds. When there are no other deviations, (25) is a sufficient condition for this strategy to be S-NM-E. Note that there is a critical pre-condition for this argument to apply: there must be sufficient monitoring such that all players involved in the punishment are informed about the deviation, for otherwise they cannot simultaneously switch to the MP strategy.

In an asymmetric AS-NM-E, groups of players which are identical *ex ante*, play different strategies *ex post*, which may yield vastly different payoffs to each group. We refer to the strategy profile for a member of a particular group as his *role*. Since these strategies are BP, it has to be the case that if i is forced to play a role with a lower payoff, then he cannot profitably switch to another role with a higher payoff. If i were to take actions as if he played a different role, other players would take actions to punish him, which would make i worse off.

In this setting, consider a society with two roles, soldiers s and farmers f. Suppose that soldiers receive a higher equilibrium payoff than farmers, $U_{\text{AS}-\text{NM}s} > U_{\text{AS}-\text{NM}f}$, and that there exists a deviation for a soldier, such that $U_d > U_{\text{AS}-\text{NM}s}$. The AS-NM$s$ strategy of soldiers demands them to treat the deviant as if he were a farmer starting from the next round, promoting another farmer to become soldier and replace the deviant. Since $U_{\text{AS}-\text{NM}s} > U_{\text{AS}-\text{NM}f}$, this farmer will gladly accept this promotion. Since the soldiers were able to prevent a farmer from pretending to be a soldier, the new group of soldiers—the old group minus the demoted soldier plus the promoted farmer—will jointly be able to prevent the demoted soldier to deny his demotion. Analogous to (25), we obtain a necessary condition for this deviation to be unattractive to soldiers:

$$\rho \left(U_d - U_{\text{AS}-\text{NM}s} \right) \leq (1-\rho) \left(U_{\text{AS}-\text{NM}s} - U_{\text{AS}-\text{NM}f} \right). \tag{26}$$

Again, (26) is a sufficient condition for the AS-NMs and AS-NMf strategies to be BP-E—with the additional requirement that there must be sufficient monitoring such that all other soldiers are informed about the deviation of one co-soldier. Note that farmers remain completely passive in this punishment, so they do not have to be informed about the soldier's deviation.

In fact, equation (25) and (26) are all we need from the repeated game structure in the remainder of the paper. They apply at all t. Hence, we omit the subscript t in what follows.

5.3 The Single Chiefdom Equilibrium SC-E

The analysis of the ST-E has revealed its major weakness: the requirement that all tribesmen monitor all fellow tribesmen implies that the tribal society runs into a size limit. The higher N the larger the share $\mu = \chi (N-1)$ of the initial endowment that has to be invested in wasteful monitoring. At some point, abandoning the cultivation of grain altogether and switching to the outside option of tubers is more efficient than sustaining the tribal social structure.

Following the ST-E discussed in the previous section, the Single Chiefdom Equilibrium SC-E discussed here is the second pivotal equilibrium. The SC-E improves on the ST-E by economizing on monitoring by largely centralizing it in the hands of one player: the chief. However, this institutional solution comes at a cost: the information monopoly of the chief will grant leverage to extract substantial surpluses from the majority of the population.

Proposition 4: the Single Chiefdom Equilibrium SC-E
Let

$$N > \frac{2\mu(1-\rho) - \beta\rho + 10}{\beta(1-\rho)}. \tag{27}$$

There are three roles:

1. 1 chief c,
2. 5 soldiers s,
3. $N-6$ farmers.

The following set of strategies for each role is AS-NM-E:

Pre-Investment Phase: If a deviant soldier d has deviated in the Robbery Phase of the previous round by attacking another player without being ordered to do so by the chief c, or by not attacking another player while being ordered to do so by c, d is demoted to f while an f is promoted to s. And c sends messages accordingly.

Investment Phase:

$$\begin{aligned} A_{\mathrm{SC}c} &= B_{\mathrm{SC}c} = 0, \qquad M_{\mathrm{SC}c} = N-1, \\ A_{\mathrm{SC}s} &= 1-4\chi, \qquad B_{\mathrm{SC}s} = 0, \qquad M_{\mathrm{SC}c} = 4, \\ A_{\mathrm{SC}f} &= B_{\mathrm{SC}f} = M_{\mathrm{SC}f} = 0; \end{aligned}$$

where c monitors all other $N-1$ players, and each s monitors the 4 other soldiers.

Pre-Taxation Phase: If a soldier d has deviated by setting $A_d \neq A_{\mathrm{SC}s}$, then c orders a similar demotion and promotion as discussed for the Pre-Investment phase.

Taxation Phase: All f-s pay a tax $T_{\mathrm{SC}fc} = \frac{1}{2}\beta$ to c.

Wage Payment Phase: The chief c pays a wage $W_{\mathrm{SC}cs} = \left(1 - \frac{1}{2}\beta\right) / (1-\rho)$ to all s-s.

Robbery Phase: Along the equilibrium path, s-s defend the chief: $I_{sc} = -1$. Other players have not invested in arms along the equilibrium path, so their attacking plans are irrelevant.

If c has deviated in the Wage Payment Phase by not paying wages to one or more s-s, all s-s observe this since they monitor each other. They will then all attack c: $I_{sc} = 1$.

If a soldier d has deviated in the Investment Phase by setting $A_d \neq A_{\mathrm{SC}s}$, c and all other s-s observe this since c monitors everybody else and s-s monitor each other. In response c sends a message to all s-s to order two other s-s to attack d: $I_{sd} = 1$, and he orders d to be demoted to farmer f, while an f is promoted to s as replacement for the deviant. If d has indeed deviated, the other s-s will follow this order.

If an f has deviated in the Investment Phase by setting $A_{\mathrm{SC}f} > 0$ or in the Taxation Phase by not paying taxes, c observes this since he monitors everybody else. In response, c sends a message to all s ordering one s to attack this f: $I_{sf} = 1$.

Out-of-equilibrium beliefs: all players believe that all other players will take the actions specified above for any deviation by any player. In particular, s-s believe that other s-s will follow orders of c, unless any s does not receive the correct wage without a prior deviation by this s. c believes that all soldiers will follow his orders as long as he pays them the correct wages, except when an s deviates. All f-s believe they will be attacked if they invest in arms or do not pay taxes.

The payoffs read

$$\begin{aligned} U_c &= 1 - \mu - 5\frac{1-\frac{1}{2}\beta}{1-\rho} + (N-6)\tfrac{1}{2}\beta, \qquad (28)\\ U_s &= \frac{1-\frac{1}{2}\beta}{1-\rho}, \\ U_f &= 1 - \tfrac{1}{2}\beta. \end{aligned}$$

Proof:

Since only s-s invest in arms they are the only group that can deviate in the Robbery Phase. Since a round ends after this phase, these deviations are not punishable within the current round and require the use of the repeated game structure. In contrast, deviations in other phases can be punished in the current round and solved by using backward induction. We start with this part of the proof.

Wage Payment Phase: only c can renege on his obligation, by not paying $W_{\mathrm{SC}cs}$ to all s-s. Since all s-s monitor each other, they observe this deviation. If so, jointly attacking c in the Robbery Phase strictly dominates all other alternative actions, making this deviation unattractive to c.

Taxation Phase: only f-s can renege on their obligation, by not paying $T_{\mathrm{SC}fc}$ to c. Since c monitors everybody, he observes this deviation. His message to order one s to attack the deviant farmer f is sent to all s-s, so that they can all observe this message and can monitor whether this s has followed the orders of c. Since all f's grain will be robbed, the best f can achieve is U_{OO} by setting $B_f = 1$, which is weakly dominated by the equilibrium payoff since $U_{\mathrm{SC}f} = 1 - T_{\mathrm{SC}fc} = 1 - \frac{1}{2}\beta = U_{\mathrm{OO}}$.

Investment Phase: c can deviate by not monitoring some players ($M_{\mathrm{SC}c} < N-1$). These players observe this. They will benefit from that: f-s will not pay taxes in the Taxation Phase; s-s will rob an f in the Robbery Phase. The payoff of this deviation is dominated by the payoff on the equilibrium path.

The critical deviation by a deviant soldier d is setting $A_d = 1$ rather than $A_{\mathrm{SC}s} = 1 - 4\chi$. Since c monitors everybody, he will observe this. He will order d to be demoted to an f immediately after the Investment Phase, implying that d receives no wage in the Wage Payment Phase and looses the surplus $U_{\mathrm{SC}s} - U_{\mathrm{SC}f}$ in future rounds. Furthermore, c will order two s-s to attack d. These s-s will accept this order, because they will otherwise be punished for not following c's orders in the next round by c.

The deviating soldier d has three options to benefit from his extra investment in arms in the Robbery Phase: he can attack either an f, or another s or c. Since the deviant is attacked by 2 other s-s, his planned attack will not succeed. Consider in particular robbing c (which is attractive because of the high potential booty: $G^W_{\mathrm{SC}c} > G^W_{\mathrm{SC}s} > G^W_{\mathrm{SC}f}$). This will not succeed since the latter is protected by 4 other s-s. These s-s monitor each other and will therefore observe d's deviation. Since they belief that 3 other s-s will follow c's orders, no other s can gain by joining d in Robbery Phase by attacking c since even then 3 other s-s remain to defend c.[9]

This completes the backward induction part of the proof. We now turn to the potential deviation by a deviant soldier d in the *Robbery Phase*. Again, d has three options: attacking an f, another s, or c. Attacking c is useless, since he is defended by 4 other s-s. Attacking s is useless, since they are equally armed, so the attack will fail. The critical deviation is therefore for d to attack an f. The punishment invokes condition (23). Since the additional payoff of d in the current round above his equilibrium payoff is equal to $G^W_{\mathrm{SC}f} = U_{\mathrm{SC}f}$, we have $U_d = U_{\mathrm{AS-NM}s} + U_{\mathrm{SC}f}$. Applying (23) yields $U_{\mathrm{SC}s} \geq U_{\mathrm{SC}f} / (1-\rho) = W_{\mathrm{SC}cs}$, which holds.

$U_{\mathrm{SC}c}$ satisfies:

$$U_{\mathrm{SC}c} = 1 + (N-6)\, T_{\mathrm{SC}fc} - 5 W_{\mathrm{SC}cs} - \mu.$$

This, and substitution of the expression for $T_{\mathrm{SC}fc}$ and $W_{\mathrm{SC}cs}$ yields (28). The payoffs for each role must be higher than U_{OO}. For f and s this condition clearly holds. For c, $U_{\mathrm{SC}c} \geq U_{\mathrm{OO}}$ yields (27).[10] ■

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[9] Note that the attack of two s-s on d ordered by c effectively contributes to the defense of c, since it neutralizes the arms of d due to the Priority of Self-defense.

[10] We have not formally checked whether c might deviate by ordering his 5 s-s to attack one f each with booty $G^T_f = U_{\mathrm{OO}}$, announcing that he will subtract this booty from his wage payment $W_{\mathrm{ST}cs}$ to soldiers. The s-s might be willing to follow this order (depending on out-of-equilibrium beliefs), since the booty offsets the lower wage and c would gain 5 U_{OO} (the saved wage payments

The SC-E allows in several ways for improvements in Hicks-Kaldor efficiency compared to the CA-E and ST-E.

First, robbery by the actual use of violence—as in CA-E—is replaced by taxation enforced by the threat of using violence. This yields an efficiency gain in using arms, since the same arms can be used to discipline multiple farmers.

Second, the introduction of a monopoly of violence of the dominant coalition of the chief and his soldiers disallows farmers wearing arms. Again, this yields an efficiency gain, since it eliminates the incentives for an arms race.

These two innovations can only be implemented by using the repeated game structure. The dominant coalition has to commit itself not to abuse its monopoly of violence for robbing farmers beyond the committed level of taxation. This commitment is only credible because the temptation to deviate is checked by the threat of losing access to the spoils of future tax revenues—directly in case of the chief, and to wage payments deriving from tax revenues in case of the soldiers.

Next, there is a large efficiency gain in monitoring. In the ST-E, everybody monitors everybody, so that total monitoring cost are μN. This imposes a limit on tribe's viable size. In the SC-E, only the chief monitors everybody and a five soldiers monitor each other, reducing the cost of monitoring to $\mu\left(1+\frac{20}{N-1}\right)$. Taking the monitoring cost per person χ rather than μ as a constant, this imposes an upper limit on the viable size of chiefdoms, namely the chief's span of control χ^{-1}. However, this limit is far less stringent than in a ST-E.

The final innovation is the separation of roles between monitoring and investing in arms, or phrased differently, between political decision making by the chief and the actual use of violence by soldiers. Not soldiers, but the chief is the most powerful and therefore richest person in society. This resonates with the emphasis placed by North et al. (2009) on the *capacity of exerting organized violence* as the defining condition of elites on societies.

For the evaluation of the Hicks-Kaldor efficiency of both equilibria, we have to compare the mean payoff $\overline{U}_{\text{SC}}$ to U_{ST}. $\overline{U}_{\text{SC}}$ is equal to the total investment in grain divided by N: $\overline{U}_{\text{SC}} = 1-(\mu+5)N^{-1}$. Using (20), we obtain:

$$\overline{U}_{\text{SC}} - U_{\text{ST}} = \mu\left(1-\frac{1}{N}\right) - \left(\frac{5}{N} - \frac{1-\gamma^{-1}}{N-1}\right). \tag{29}$$

The first positive term is the saving on monitoring cost, the second negative term is the additional cost of arms. The investment in arms is larger in a SC-E than a

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for his 5 s-s). A necessary condition to make this deviation unattractive, invoking equation (25) and using U_{OO} as c's payoff after this deviation (which is a lower bound on his actual payoff after this deviation), reads: $U_{\text{SCc}} > \rho(U_{\text{SCc}} + 5U_{\text{OO}}) + (1-\rho)U_{\text{OO}}$, or equivalently: $(1-\rho)U_{\text{SCc}} > (1+4\rho)U_{\text{OO}}$. This condition will be satisfied for sufficiently high N (such that $U_{\text{SCc}} >> U_{\text{OO}}$) and sufficiently low ρ. Implicitly, our proof takes for granted that this condition is satisfied.

ST-E due to the necessity to defend the chief's high tax revenues from robbery. He is particularly vulnerable to attacks by the well-armed soldiers who are supposed to protect him. He therefore hires five of them, countering the threat of one of them deviating and another jumping on the bandwagon. Apart from the cost of monitoring their colleagues, soldiers are fully armed, so that they cannot be outflanked by a more heavily armed deviant.

The monopoly of violence imposes a fixed cost on society: five soldiers and the monitoring by the chief. For sufficiently high N, see (29), this fixed cost is small relative to the total grain production and the mean payoff is higher in a SC-E than a ST-E. However, this higher mean payoff comes at a price for the large majority of farmers: their payoff is just ε above U_{OO}. The soldiers and in particular the chief are the beneficiaries of the Hicks-Kaldor efficiency gain. Farmers, to the contrary, are worse off because they earn the mean payoff in the ST-E but only the outside option in the SC-E. A transition from a ST-E to a SC-E is therefore not a Pareto improvement.

6 Intergroup conflicts

In the equilibria considered so far, all players form a single society with internal monitoring in case of a ST-E and a single hierarchical structure in case of a SC-E. However, while the SC-E yields an efficiency gain in monitoring cost compared to the ST-E, even the SC-E faces a size limit given that the span of control of the chief is limited to $(N-1) \leq \chi^{-1}$. In response to this, it is natural to investigate equilibria of multiple groups, as players split up to reduce the cost of monitoring all other group-members.

There is a second motivation for studying equilibria with multiple societies. Being a farmer in a SC-E is an undesirable ordeal. It is unlikely that a dominant coalition can easily subdue players to be part of a society that yields them a payoff of only ε above their outside option. Groups of players can be expected to resist incorporation into a chiefdom. History indeed confirms that chiefdoms (and later states) have been surrounded by tribes and bands that have resisted incorporation in the chiefdom's hierarchical structure, often successfully and for long stretches of time.

This section therefore analyses equilibria where two societies live next to each other, either two tribes or a tribe and a chiefdom. These societies have to manage two violence problems: internal robbery between members of the own group, and external robbery, where one group attacks the other. We study equilibria both where two groups are at war, and where they peacefully coexist under the threat of a transition to intergroup violence.

6.1 The equilibria with multiple tribes

This subsection considers two equilibria, the Warring Tribe Equilibrium WT-E and the Peaceful Tribe Equilibrium PT-E. We start with the WT-E, which is a mixed strategy equilibrium where two tribes randomly decide between attacking and not attacking the other tribe. An exact analytical characterization of the WT-E is infeasible as it requires solving higher order polynomials. We therefore apply first order Taylor expansions for larger N. By this approach, some terms in the payoff functions can be ignored as they are of higher order, which allows an analytical solution of the remaining system.

Proposition 5: the Warring Tribe Equilibrium WT-E
Let N be even and define $L := N/2$, and let:

$$\begin{aligned} \gamma^{-1} &> 1-\frac{1}{2}\beta^{-1}-\beta\mu^2/8+\mathcal{O}\left(N^{-1}\right), && (30)\\ \gamma^{-1} &< 1-\frac{1}{2}\mu+\mathcal{O}\left(N^{-1}\right), && (31)\\ 0 &< \left(1+\frac{1}{2}\beta\gamma\right)^2+(1-\beta)\gamma\left(1-\frac{1}{2}\beta\gamma\right)\mu+\mathcal{O}\left(N^{-1}\right). && (32) \end{aligned}$$

There are two tribes of equal size L; each player belongs to one of them. Each tribe has one big man b who coordinates behavior of his tribe's members, but plays the same strategy along the equilibrium path and receives the same payoff.

Define:

$$\mathrm{P}_{\mathrm{WT}} := 1-\frac{1}{2}\mu\beta-\sqrt{\left(1-\frac{1}{2}\mu\beta\right)^2-2\beta\left(1-\gamma^{-1}-\frac{1}{2}\mu\right)}+\mathcal{O}\left(N^{-1}\right). \qquad (33)$$

Then the following strategy played by all i is MP-E:

Prior to Investment Phase: each tribe's b decides randomly what actions his tribe's members will play that round, either to attack Y with probability P_{WT} or to defend X with probability $1-\mathrm{P}_{\mathrm{WT}}$ and sends a message to his fellow tribesmen accordingly.

If b chooses X:

Investment Phase:

$$A_{\mathrm{WT}X}=\mathcal{O}\left(N^{-1}\right), \qquad B_{\mathrm{WT}X}=\mathrm{P}_{\mathrm{WT}}/\beta, \qquad M_{\mathrm{WT}X}=L-1. \qquad (34)$$

i monitors all his fellow tribesmen.

Taxation and Wage Payment Phase: there are no tax and wage transfers.

Robbery Phase: along the equilibrium path, i defends himself ($I_{ii}=-1$); only if one of his fellow tribesmen d has deviated on either $A_{\mathrm{WT}X}$ or $M_{\mathrm{WT}X}$ in the Investment Phase, i plans to attack d ($I_{id} = 1$).

If b chooses Y:

Investment Phase:

$$A_{\mathrm{WT}Y} = A_{\mathrm{WT}X} + \gamma^{-1} = \gamma^{-1} + \mathcal{O}\left(N^{-1}\right), \qquad B_{\mathrm{WT}Y} = 0, \qquad M_{\mathrm{WT}Y} = 0; \tag{35}$$

Taxation and Wage Payment Phase: there are no tax and wage transfers;

Robbery Phase: b sends a message to each of his fellow tribesmen i which member j of the other tribe i is supposed to attack —avoiding that any two tribesmen attack the same member of the other tribe and have to share to booty. Accordingly, i attacks j: $I_{ij} = 1$.

Out-of-equilibrium beliefs: for action X, they are the same as in the ST-E. For the action Y, they are irrelevant.

The expected payoff is

$$U_{\mathrm{WT}} = 1 - \gamma^{-1}\mathrm{P}_{\mathrm{WT}} - \frac{1}{2}\mu\left(1 - \mathrm{P}_{\mathrm{WT}}\right) - \frac{1}{2}\beta^{-1}\left(1 - \mathrm{P}_{\mathrm{WT}}\right)\mathrm{P}_{\mathrm{WT}}^{2} + \mathcal{O}\left(N^{-1}\right). \tag{36}$$

Proof: see Appendix A.2.

In this equilibrium the big men of both tribes choose each round independently between attacking the other tribe (Y, with probability P_{WT}) or not attacking (X, with probability $1 - \mathrm{P}_{\mathrm{WT}}$). Each round has therefore four possible outcomes: both tribes attack (probability $\mathrm{P}_{\mathrm{WT}}^{2}$), one tribe attacks another or the other way around (both with probability $\mathrm{P}_{\mathrm{WT}}\left(1 - \mathrm{P}_{\mathrm{WT}}\right)$), or both tribes do not attack each other (probability $\left(1 - \mathrm{P}_{\mathrm{WT}}\right)^{2}$). Since big men have no enforcement power, their choice has to be self-enforcing.

When b chooses X, all tribesmen monitor each other. They will punish any deviation in the Investment Phase by jointly attacking the deviant in the Robbery Phase, similarly to the ST-E. By a similar logic as in that equilibrium, an investment in arms of order $\mathcal{O}\left(N^{-1}\right)$ is enough, since the coordinated attack of all tribesmen on a deviant is a sufficient deterrent even if each individual tribesman invests only a small amount in arms. The costly part of deterrence is therefore not the investment in arms, but in monitoring of other tribesmen. This costly internal monitoring comes with one unintended beneficial consequence: it reduces the share of the endowment that can be invested in grain, thereby reducing the attractiveness for the other tribe to choose Y.

This is also the reason why we can provide closed form solutions for the equilibrium conditions up to order $\mathcal{O}\left(N^{-1}\right)$, since we can ignore the investment in arms needed for internal deterrence. Only by ignoring these higher order terms can we obtain manageable expressions.

When choosing Y, tribesmen are not supposed to monitor each other. Hence, internal deviations remain undetected. However, monitoring would not resolve the problem of an internal deviation within an attacking tribe, since tribesmen have the opportunity to rob a member of the other tribe, which yields a greater booty than punishing a deviant of the own tribe. The threat of internal punishment is therefore non-credible anyway. The only solution for the big man is to make joining the attack a self-enforcing choice since its payoff exceeds the payoff of a deviation. In this way, attacking the other tribe provides protection of own investment in grain against robbery by fellow tribesmen. Hence, only the defending tribes harvest tubers. This is the common enemy effect, see De Jaegher (2022): the existence of a common enemy contains internal violence. Making joining the attack self-enforcing therefore requires the big man to reduce the probability P_{WT} of choosing Y, so that the other tribe invests substantially in grain instead of substituting away to tubers, thereby increasing the booty of robbery.

This equilibrium requires a number of conditions on the admissible parameter values, see (30)–(32). If one of these conditions is violated, this strategy is not MP-E, since either the probability P_{WT} is not defined, or the defending tribe's investment in grain is too low to justify the attacking tribe's investment in arms, or $U_{\mathrm{WT}X}$ is less attractive than the outside option.

The eventual expression (36) for the expected payoff U_{WT} is highly intuitive: the initial endowment one minus the probability of playing Y times the cost of investment in arms γ^{-1} in that case and minus the probability of playing X times the cost of monitoring and the efficiency loss of tubers instead of grain. Note that the booty does not affect the expected payoff: robbery is purely redistributive between the actions sets from the payoff when playing X and when playing Y, and therefore does not affect the expected payoff of both actions.

The WT-E is highly Hicks-Kaldor inefficient, since substantial resources are spent on investing in arms to rob the other tribe when playing Y and investing in tubers to protect against robbery by the other tribe when playing X. The Peaceful Tribe equilibrium PT-E uses the inefficiency of the WT-E as a threat to achieve a more cooperative equilibrium. Any deviation by either tribe of the PT-E will be punished by both tribes collectively switching to the WT-E which is bad for everybody.

Proposition 6: the Peaceful Tribe Equilibrium PT-E

As in Proposition 5, let N be even and let $L := N/2$. Furthermore, let the conditions

(30)–(32) for a WT-E hold and let $\mathrm{P_{WT}}$ satisfy (33). Moreover,

$$\rho\left(1-\gamma^{-1}\right) \leq \gamma^{-1}-\frac{1}{2}\mu+\frac{1}{2}\beta^{-1}\left(1-\mathrm{P_{WT}}\right)\mathrm{P_{WT}}. \tag{37}$$

Again, there are two tribes of equal size; each player belongs to one tribe. Each tribe has a big man b who can coordinate the actions of his fellow tribesmen.

The following strategy played by all members of each tribe is S-NM-E:

All members of both tribes play the same strategy as in ST-E, unless at least one player attacks a member of the other tribe. This is observed by all players, as all players monitor their fellow tribesmen. Hence, the tribe of the robber observes his robbery, while the tribe of the robbed observes that one of their fellow tribesmen is robbed and not by one of their own tribesmen. Then all members of both tribes switch collectively to the WT-E.

The expected payoff is

$$U_{\mathrm{PT}}=1-\frac{1}{2}\mu+\mathcal{O}\left(N^{-1}\right). \tag{38}$$

Proof:
The equilibrium payoff U_{PT} of a tribe of size L is the same as U_{ST}, see (20), setting $N \to L$ and using $L/N=\frac{1}{2}+\mathcal{O}\left(N^{-1}\right)$. This yields (38). The critical deviation is a big man who suggests to his tribesmen to attack the other tribe. By setting $A_d=\gamma^{-1}$ they will rob the full grain budget G_{PT} of the other tribe:

$$U_d=2-\gamma^{-1}-\frac{1}{2}\mu+\mathcal{O}\left(N^{-1}\right).$$

For a S-NM-E, (25) must hold. Hence:

$$U_{\mathrm{PT}} \geq \rho U_d+(1-\rho)\,U_{\mathrm{WT}}$$

Substitution of U_d above and using (36) and (38) yields (37).■

Proposition 6 shows that two tribes that live at war with each other in a WT-E can reach a PT-E using the threat of the return to the WT-E as deterrence against deviations of the other tribe, provided that condition (37) holds. A necessary but insufficient condition for this is that the cost of monitoring is smaller than the cost of violence (both the cost of arms and substituting away to tubers):

$$\frac{1}{2}\mu<\gamma^{-1}+\frac{1}{2}\beta^{-1}\left(1-\mathrm{P_{WT}}\right)\mathrm{P_{WT}}.$$

If this condition does not hold, $U_{\mathrm{PT}} \leq U_{\mathrm{WT}}$ and aspiring peace does not pay of anyway. If $U_{\mathrm{PT}} > U_{\mathrm{WT}}$, (37) always holds for a sufficiently small ρ, or conversely, for a sufficiently large weight of the future.

Note that the big man plays a negative role in the PT-E. Without a big man, each tribe would punish deviant members by a joint attack. Hence, tribes would not have to worry about an attack of a deviant from the other tribe since the other tribe would deal with this deviant internally. Monitoring the other tribe is unnecessary. The coordination by the big man enables tribesmen to coordinate their attack, making a collective deviation by the other tribe a realistic threat.

The monitoring requirements of the PT-E become large for large L. Given the lack of internal monitoring when a tribe attacks in the WT-E, for large enough N, $U_{\mathrm{PT}} < U_{\mathrm{WT}}$ and the PT-E is no longer attainable.

The PT-E works fine for two tribes, since the internal monitoring of each tribe will reveal any robbery by a member of the other tribe on a fellow tribesmen by default, since a monitored robbery of fellow tribesmen by an un-monitored player must be done by a member of the other tribe, as all other players in the own tribe are monitored. Hence, everybody can join in the punishment of a transition to the PT-E. With three tribes, the third tribe would not observe the attack and would therefore not be able to join in the transition in the next round, while the tribe that has been robbed does not know which other tribe was to blame for the robbery. Accordingly, the switch from PT-E to WT-E does not follow immediately after a deviation and the deviating tribe can potentially profit for multiple rounds (depending on the number of tribes) from attacking a different tribe each round before all tribes are attacked and, thus, aware of the deviation.

Combined, the monitoring costs in the PT-E and the slow transition from the PT-E to WT-E when the number of tribes increases, lead up to an important conclusion. They imply that the curse of size that hampers the feasibility of extending tribal societies is not easily remediated by splitting tribes in multiple smaller tribes and relying on the PT-E to contain inter-tribal violence. This makes the transition to a chiefdom more likely.

6.2 The Warring Chiefdom Equilibrium WC-E

The equilibrium that we consider in this subsection features a tribe and a chiefdom. This equilibrium is particularly relevant from a historical perspective since tribes and chiefdoms have existed next to each other for long stretches of time.

Proposition 7: the Warring Chiefdom Equilibrium WC-E

Let N be even and let $L := (N-6)/2$. There is one chiefdom with one chief c, 5 soldiers s, and L farmers f and one tribe with L members t. Let:

$$\beta \leq \frac{2\gamma(\gamma-1)}{6\gamma-5}. \tag{39}$$

The following set of strategies for each player is AS-NM-E.

The tribe:

Investment Phase:

$$A_{\mathrm{WC}t} = 0, \qquad B_{\mathrm{WC}t} = (\gamma-1)\,\gamma^{-1}, \qquad M_{\mathrm{WC}t} = 0. \tag{40}$$

Taxation and Wage Payment Phase: t-s make no tax and wage transfers;

Robbery Phase: since $A_{\mathrm{WC}t} = 0$, t-s cannot attack others.

The chiefdom:
All roles play a similar strategy as in the SC-E. We discuss only the differences for each role:

Investment Phase: contrary to the SC-E, $A_{\mathrm{SC}f} = \beta\,(\gamma-1)\,\gamma^{-2}$.

Taxation Phase: f-s pay c the same tax as in the SC-E.

Wage Payment Phase: c pays s-s the same wage as in the SC-E.

Robbery Phase: c orders each f which t to attack so that no two f-s attack the same t: $I_{ft} = 1$. f-s follow these orders; any deviation by f-s or s-s in previous stages will be punished in a similar way as in the SC-E.

Out-of-equilibrium beliefs: beliefs in the chiefdom are the same as in the SC-E.

The payoffs of s and f are the same as in the SC-E. The payoffs of c and t read:

$$\begin{aligned} U_{\mathrm{WC}c} &= 1-(L+5)\chi+\tfrac{1}{2}\beta L-5\frac{1-\frac{1}{2}\beta}{1-\rho}, \\ U_{\mathrm{WC}t} &= 1-\frac{1}{2}\beta\frac{\gamma^2-1}{\gamma^2} \end{aligned} \tag{41}$$

Proof: see Appendix A.3.

The WC-E relies on permanent deterrence of the tribe by the chief ordering his farmers to attack tribesmen every round. The f-s' attack serves a double protective purpose. First, the necessary investment in arms reduces f-s' robbable grain $G^{W}_{\mathrm{WC}f}$, thereby making an attack by a t on f less attractive. Second, it undermines the potential attacking power of a t, since he first has to defend against the attack of one f before he is able to counterattack another f. The latter f is forced to use his force $A_{\mathrm{WC}f}$ to self-defend rather than attacking another f, in accordance with the rule of

The Priority of Self-defense. One time the cost $A_{\mathrm{WC}f}$ is recouped by blocking the f's robbery, the second time is not.

This deterrence requires some precise conditions to hold. Each f's attack strength $A_{\mathrm{WC}f}$ is set such that t-s substitute away from grain to tubers $B_{\mathrm{WC}t} = \frac{\gamma}{\beta} A_{\mathrm{WC}f}$ such that the remaining investment in grain satisfies $G_{\mathrm{WC}t} = \gamma^{-1}$, implying $\gamma G_{\mathrm{WC}t} = 1$. By this condition, both c ordering f-s' to attack t-s' and t-s' not engaging in self-defense are both just weakly dominated by the alternatives of not attacking and engaging in self-defense respectively. Remarkably, the chief's payoff is exactly the same in the WC-E as in the SC-E, compare (41) and (28) (adjusting for smaller size of the chiefdom with L rather than $N-6$ farmers). The cost of farmers' investment in arms is exactly offset by the booty from robbing the tribe.

Equally remarkable is that this equilibrium does not require internal monitoring by the tribe. If a t attacks another t, he has to share his booty with a f who is also attacking t's victim. The unilateral attacks by f-s just break even. So does a unilateral attack by t. A multilateral attack of a f and a t implies that the booty has to be shared between them, implying that the booty does not justify the investment in arms. No monitoring within the tribe is therefore required. Depending on the monitoring cost μ, the WC-E therefore might yield tribesmen an even higher payoff than the ST-E. This is the case if, see (20),

$$U_{\mathrm{WC}t} \geq U_{\mathrm{ST}} \Leftrightarrow \beta \frac{\gamma^2 - 1}{2\gamma^2} \leq \mu + \mathcal{O}\left(N^{-1}\right).$$

The WC-E is fragile. We do not allow the tribe to appoint a big man b to coordinate a collective attack on c. If we would, the equilibrium falls apart. The potential booty per tribesman is dominated by c's tax revenues, see (41):

$$G^{W}_{\mathrm{WC}c}/L = U_{\mathrm{WC}t}/L = T_{\mathrm{WC}fc} + \mathcal{O}\left(N^{-1}\right).$$

This makes c an attractive target. Though c is well protected by five soldiers that invest their entire endowment in arms except for a small investment in monitoring. Still, the soldiers cannot withstand a collective attack by all tribesmen: s-s' combined strength, $5A_{\mathrm{WC}s} = 5\left(1 - 4\mu/N\right)$ is of order $\mathcal{O}\left(N^0\right)$, while t-s' combined strength is of order $\mathcal{O}\left(N\right)$. For large N, the latter always dominates and c's only chance of withstanding the attack is to order his f-s' to counterattack. A coordinated attack pays off for the tribesmen if the booty plus the cost of overcoming f-s' counterattack exceeds the cost per tribesmen of the investment in arms, $A_{\mathrm{WC}t} = A_{\mathrm{WC}f} + \mathcal{O}\left(N^{-1}\right)$, where the second term covers the arms needed to counter s-s' combined defense of c

and the net strength $S_c = \gamma^{-1}$ needed to rob c:

$$T_{\mathrm{WC}fc} + A_{\mathrm{WC}f} + \frac{1}{2}\beta B_{\mathrm{WC}t}^2 + \mathcal{O}\left(N^{-1}\right) \geq A_{\mathrm{WC}f}.$$

This condition is satisfied since $A_{\mathrm{WC}f}$ cancels and the other two terms are positive.

In order to protect himself, c has to deter the big man from proposing an attack by ordering his fs' to invest more in arms by setting $A_f > A_{\mathrm{WC}f}$. If c sets the norm on A_f sufficiently high, this deterrence will probably be successful. However, while deterrence is cost neutral in the WC-E (the booty exactly offsets the cost of arms), it will be costly when c picks a higher level of A_f. Since c must give his f-s' their outside option anyway (stated differently: c is the residual claimant, who will bear the burden of any additional costs and revenues), this cost comes at his expense, by means of lower tax revenues. This raises the question whether c's deterrence will be credible. He is likely to scale down A_f as soon as the threat of a tribal attack has vanished. Probably, there is a mixed strategy equilibrium, but that is hard to specify. Conflicts between chiefdoms and tribes might therefore be long lasting, unstable and hard to contain.

7 Insights on the transition from tribes to chiefdoms

Even though we do not offer a formal model of transition from one set of institutions to another, our model and the equilibria it generates still hold relevance to better explain the shift from an ideal-typical tribe to an ideal-typical chiefdom, or: the emergence of hierarchical government. More specifically, we can confront the various hypotheses formulated in the literature to explain this transition (for overviews of the different historiographical strands and main hypotheses: Mayshar et al. (2022); Feinman (2017), and Section 2) with the logic of the model. These hypotheses roughly fall apart into the following categories:

- changes in agricultural techniques and the emergence of arable agriculture;
- the drive for enhancing welfare and solving collective action problems;
- the introduction of new types of weaponry;
- changing ideologies;
- population growth.

Before looking at these factors, it is important to note again that this transition was not a linear event. Our model helps to understand how chiefdoms could fail again and how chiefdoms and tribes could exist for a long time alongside chiefdoms without being immediately absorbed into them. Tribes were able to manage the threat of internal violence well. Most clearly, this applies when compared to a hypothetical situation in which tribes would not exist and which we have modeled as Complete Anarchy. This situation, which is not meant to be historical representation but can be used as a benchmark, was typified by relatively high investments in tubers and arms, and thus likely generated lower utility compared to tribes. However, the equilibria discussed in Section 6 show that tribes could function well in competition with other tribes, but that the Peaceful Tribes equilibrium is sensitive to size—due to internal monitoring costs and the slow spread of information when the number of tribes increases. Also, we show how chiefdoms are strictly welfare decreasing from the perspective of most of its participants. Absorption of tribes into chiefdoms was therefore not an automatic or attractive process. Furthermore, the Warring Chiefdom equilibrium does not hold when the tribe has a big man that coordinates attacks against the chiefdom. Jointly, these findings of our model suggest tendencies both towards hierarchy as well as towards collapse of hierarchy.

Going over the factors listed above, which are mentioned or hypothesized in the literature as having caused the transition to chiefdoms and scrutinizing them in the light of empirical evidence in combination with the model developed here, it is clear that most of these factors have been a necessary but not sufficient condition for this transition or even have been a result rather than a cause of this transition. Also, it is clear, in line with the emerging consensus in the anthropological and archaeological literature, that there has not been one 'prime mover' in the process (Feinman, 2017) but rather a combination of factors.

First, there is agricultural innovation, which is often mentioned among the factors that drove the transition to chiefdoms, sometimes in combination with climate change. Both the empirical evidence and the model show that the shift to sedentation and grain cultivation were a necessary condition for the transition to chiefdoms, as grain made the extraction of surpluses and storage of resources possible, but not a sufficient condition (Scott, 1998, 2017). This is empirically evidenced by the long period between the rise of arable agriculture and the transition and also by the longevity of tribes that did concentrate on grain production (see previous Sections 1 and 2) and their ongoing existence alongside chiefdoms. It is also in line with our model, which generates stable equilibria with grain-producing tribes. Further agricultural innovation, as with the introduction of draft animals, has likely pushed up inequalities (Bowles and Fochesato, 2024) but accumulation of animals and land required property rights and their protection, which points to the direction of causality.

The role of new weaponry is more difficult to disentangle as a factor. Monopolization of violence capacity in the hands of a small, dominant coalition is a main component of the shift to chiefdoms and, indeed, the period in which this shift took place saw major military innovations that facilitated monopolization. As noted above (Section 2; also Ferguson (2013)), these innovations either required more coordination or consisted of the introduction of weaponry that could more easily be monopolized, as the bronze weapons and, later, war horses and chariots. It is important to recall, however, that these major innovations in weaponry took place a longer period after the rise of chiefdoms. Even though both the costs and the enlarged possibilities for monitoring and restricting the use of this more advanced weaponry may have facilitated processes of monopolization into the hands of a subset of the population, this again seems to have been more the effect than the cause of the shift. It is also relevant here to stress that our model highlights how the chief did not build his position on his own strength or control over any weaponry but on his control over the military. Still, this new military technology was well-suited to the constellation developed in the emerging chiefdoms, with emerging hierarchy pushing demand for innovation in weaponry and making specialization in its production and use more feasible.

The emergence of a new 'chief ideology' or the religious legitimization of hierarchy is another factor often mentioned to explain the shift to chiefdoms (Gat, 2008, 223–7; Bentzen and Gokmen, 2023). This factor is not independent of our model, however. Both the Single Tribe and the Single Chiefdom equilibrium are based on self-interested behavior, but in a context of a system of social norms. The prevailing systems of norms of tribal societies and chiefdoms, however, are almost opposite. The normative system of tribes opposes any use of authority and even punishes a deviant who is claiming a leadership position with collectively executed penalties (e.g. Earle, 1978; Scheidel, 2018, 28–32; Wrangham, 2019, 158–67). At the same time, this system of collective punishment of deviants created societies where individuals continuously had to worry about collective judgments (Acemoglu and Robinson, 2019). The normative system of chiefdoms, on the other hand, rewards respect for authority and acceptance of one's role in society, thus consolidating and legitimizing inequalities. Whether these social norms are upheld by rational deliberation in the context of Bayesian beliefs about other players' responses or by evolutionary selection against players with a deviant innate normative response might be a second order question. In both cases, adjusting to the prevailing normative system is key to the individual survival in the respective types of society. From the empirical literature, we know that in chiefdoms indeed a 'chief ideology' did develop (in line with the results of the quasi-natural experiment by Bentzen and Gokmen (2023)). In the context of our model, this ideology is easy to understand as a rationalization of the Bayesian beliefs critical for upholding the equilibrium. Our model shows that the emergence of this type of beliefs is a logical implication of the constraints on society (in line with Carneiro (2002)).

The main remaining factor is population growth, which is often quoted as a driving force behind the development of hierarchy, with population growth and societal complexity pushing each other up (e.g., Diamond, 1997, 284–92). Tribes indeed run into size limits due to the intensive monitoring requirements, as is shown in the model. This also links up with empirical observations, that is: the absence of tribes with large numerical sizes—see Feinman (2013) for a nuanced discussion. Still, the preceding leads us to qualifying this factor, since we also show that, after having split up due to rising numbers, it is rewarding for tribes to peacefully co-exist, while a warring situation reduces internal monitoring costs, making larger numbers for a tribe less of a problem.

This changes when population densities further grow and territory becomes scarce. Along various lines this may reduce the incentives for tribes to resist the shift to chiefdoms. First, it makes the outside option of producing tubers less attractive, since grain yields more output per surface area, needed to feed the growing population. Second, this situation reduces surpluses since higher labor input per hectare pushes up physical output but increasingly brings output per labor unit under pressure. Further, it increases the cost of monitoring per tribesman. The resulting decline in welfare makes the shift to chiefdom for farmers less unattractive. Scarcity of territory makes splitting up of tribes more difficult, which leads to larger group sizes and higher monitoring costs in tribes. Lastly, it leads to more conflict about scarce land and more open violence and warfare. A particular aspect of this is highlighted by our model, which shows how maintaining peace between tribes becomes less easily with more than two tribes, since a violation by one tribe is only detected by the violating tribe and its victim, not by others, which made intertribal warfare more widespread. In its turn, increasing warfare and robbery make the cultivation of appropriable grain less attractive and induce a shift to tubers, which are more easily protected from robbery but yield less, thus further aggravating the problem of population pressure and leading to a vicious cycle.

Jointly, in this situation of increasing competition over land due to population pressure, we can expect the relatively beneficial tribe equilibria to become less attractive. Moreover, the situation of endemic violence and conflicts between tribes, fueled by competition over scarcer resources, made the emergence of a ‘big man’ more likely. This is in line with our model, where the Single Tribe equilibrium does not offer a role to a big man, but the Warring Tribes equilibrium does. While these ‘big men’ could also be disposed again, it may have been attractive to the tribesmen to let him fulfill a more permanent role in monitoring, as this saved monitoring costs and would be welfare-enhancing. In turn, this could lead to the big man usurping more prolonged power, perhaps in combination with some tribe managing to attain dominance over one or more other tribes. By centralizing monitoring, and thus saving monitoring costs, some tribes could sustain larger numbers, which made the acquisition of such dominance in combination with the emergence of a chief more likely.

The conjecture that this was an essential phase in the shift to chiefdoms is also in line with anthropological and archaeological literature suggesting an increasing frequency and intensity of violence and warfare just before and during the rise of chiefdoms (Carneiro, 1998). One may surmise that a shift to chiefdoms is even further sped up by geographical conditions where resources were spatially highly concentrated (Smith and Codding, 2021) or made exit for people costly or difficult, and it made the problem of population pressure more imminent. A major example is the Nile Valley which is surrounded by arid deserts, a constellation explicitly highlighted in the 'circumscription theory' in the older work by Carneiro and later on reformulated in an adjusted, more nuanced form to explain the emergence of hierarchy (Carneiro, 2012).

It is important to stress again, however, that the shift to chiefdoms was not the automatic or sole possible outcome of increasing population pressure, as it required a joint set of three institutional innovations, underpinned by associated social norms. As shown in this paper, the emergence of a chiefdom required not only the rise of a monopoly of violence imposed by a dominant coalition but also the combination with: (i) appropriation by the chief from the farmers takes place under the threat rather than the actual use of violence by the chief's soldiers, (ii) constraints on the members of this dominant coalition to prevent the abuse of their violence capacity against farmers by the threat of demotion out of the dominant coalition, and (iii) the monopolization in the hands of the chief of both monitoring and decision making on the use of violence. Moreover, and arguably forming the most important implication of the model, this control over information via his monitoring activities is what turns the chief into the dominant player—i.e., power derives from information rather than violence capacity.

Only in combination with each other these innovations worked as building blocks of a chiefdom. Without these innovations, the same situation of warfare induced by population pressure could easily have alternative outcomes. The incessant warfare could drive down population pressure through migration and/or violent deaths, or population numbers could be reduced through Malthusian preventive (infanticide) or by positive checks (disease, food crises).

While we show that a Single Chiefdom equilibrium exists in our model, the model also shows that it remains vulnerable. With the growth of the number of people in the chiefdom the prosperity of the chief grows but thereby also the attractiveness of the chief as a target for robbery by a tribe. The best strategy for chiefs in this context, according to our model, is to semi-permanently use the farmers as low armed conscripts to attack tribesmen, as an indirect means to discourage the tribe to try to rob the chief's riches. This is somewhat of a contradictory outcome: the monopoly of violence in the hands of dominant coalition protects farmers against internal violence by other members of the chiefdom, but to protect the chiefdom from coordinated external violence, the chief has no alternative than to lift the ban on wearing arms for farmers and to use them as conscripts to attack tribesmen.

These observations help us to understand the differences in chronology and nature of the shifts from tribe to chiefdom, and the fact that chiefdoms often disintegrated again (points stressed by Feinman (2017)). At the same time, our analysis still enables us to discern more general factors in the rise and ultimate dominance of chiefdoms and to shift the focus from physical and technological factors to social mechanisms.

8 Discussion and conclusion

A large body of literature has documented the transition of egalitarian tribal societies to hierarchical chiefdoms in a protracted process taking several millennia, which nevertheless occurred at different locations in the world with similar outcomes. These chiefdoms were characterized by: (i) a stratification of society based on distinct roles, (ii) a monopoly of violence by a small dominant coalition under the central command of a chief, and (iii) high inequality based on tributes paid by farmers to the chief. Due to these tributes, farmers generally were worse off than in tribal societies. This process has been non-monotonic, and tribes and chiefdoms have existed next to each other for millennia, either at war or in peace, and there have been many reversals, where chiefdoms collapse under the pressure of warring tribes. What have we learned from our model beyond these well-established facts? We see five contributions.

First, our model generates all these predictions naturally in one unifying framework that enables us to analyze both violence within the group and violence between groups. It generates distinct equilibria that look very much like either tribal societies or chiefdoms, with all the characteristics that this literature has attributed to them. That is, given a set of logical constraints on the behavior of the model's agents, the model naturally produces the expected outcomes listed above. The model thus formalizes the two main forms of societal organization at both sides of the emergence of 'government' within one and the same consistent framework.

Second, in our model, the distinct roles that individuals take in chiefdoms are not imposed by a priori assumed but innate differences between players, like in most canonical models of societal organization—e.g., the analysis of the transition to democracy by Acemoglu and Robinson (2006) and Besley and Persson (2011)—or the standard conflict models with hierarchy—e.g., Grossman and Kim (1995, 1996); Hirshleifer (1995); Konrad and Skaperdas (2012); and Dal Bó et al. (2022). In contrast, our model starts with homogeneous individuals. Different roles emerge endogenously as an equilibrium outcome, enforced by the capacity of some players to control information and thus constrain the options for cooperative play among other players. Accordingly, our model provides new insights into the existence of hierarchy and its emergence.

Third, our paper generates new predictions that can be tested either by future empirical research or by the re-evaluation of existing empirical evidence. We offer some examples. First, a comparison of the Single Tribe and the Warring Tribes equilibria shows that an external conflict can be a substitute for the internal monitoring required in the Single Tribe equilibrium to avoid internal robbery (the common enemy effect, see De Jaegher (2022)). Second, the model predicts that a chiefdom at war with a tribe does not use its elite soldiers for its conflict with the tribe. Instead, these soldiers‘ main task is to protect the chief and to punish internal deviations. Accordingly, the chiefdom relies on conscripted farmers to fight against tribesmen. To do so, farmers are required to invest a small amount of their endowment in arms and attack tribesmen under the coordination of the chief. Thirdly, our most surprising prediction is that the power of the chief does not rest upon his outstanding violence capacity, but on his information monopoly. We are aware of an anthropological reference suggesting this result (Creamer and Haas, 1985, 740), but no systematic empirical evaluation of this claim.

Fourth, our model fills a lacuna in economic theory. Previous models of conflict and endogenous peace, like Skaperdas (1992); Hirshleifer (1995); and Dixit (2011) usually have only two players and generate a diversification of roles only through innate differences between players. These models find it hard to explain why most societies have ended up in installing a monopoly of violence. In a recent application of the Skaperdas model, it is shown that for many specifications of the production and combat technology, a monopoly of violence is a Pareto inferior solution (Abramson, Awad, and Kenkel, 2022). Our model can easily explain why some faction in society favors the introduction of a monopoly of violence. The model implies that a monopoly of violence is focused on internal coercion at the hands of a group of violence specialists. In addition, these violence specialists do not offer protection from external threats, as is seen most clearly in the Warring Chiefdom equilibrium, where farmers are forced by the dominant coalition to fight off the external threat themselves. Accordingly, the monopoly of violence derived in our model is a pure protection racket. We hope that this raises follow-up theoretical research along these lines.

Finally, our model helps to understand why the rise of chiefdoms has taken several millennia. Though our model does not offer a full theory of institutional dynamics, it offers three useful insights. First, it shows that the emergence of a chiefdom requires not only the rise of a monopoly of violence imposed by a dominant coalition but also the combination with several other institutional transitions: (i) robbery by the threat rather than the actual use of violence, (ii) constraints on the members of this dominant coalition to prevent the abuse of their violence capacity against farmers, (iii) the monopolization in the hands of the chief of both monitoring (i.e., information collection) and decision making on the use violence. The critical point here is that these transitions are complements: one cannot work without the other. For example, one cannot enforce the payments of tributes under the threat of violence when soldiers abuse their power to apply violence anyway. Similarly, the threat of using violence is only effective under a monopoly of violence for the dominant coalition. The stability of chiefdoms is, however, difficult to maintain, and chiefdoms remain vulnerable to the attacks of tribes. As a second insight, we have shown one way in which chiefdoms can co-exist with tribes, as is observed in history.

Our model also shows how a chiefdom at war with a neighboring tribe cannot protect itself against the attacks of tribesmen by way of elite soldiers; it has to use a mass of conscripted farmers to ward off their attacks. Elite soldiers have a comparative advantage to defend against attacks of an individual deviant, conscripts have a comparative advantage in mass conflicts with large numbers of combatants on both sides.

A core result of the paper is that it provides a clear logic for the protracted and often volatile path to the emergence of hierarchical government. We show that the transition to chiefdoms may improve Hicks-Kaldor efficiency by economizing on monitoring, but this transition is not a Pareto improvement, as farmers become worse off. The model generates stable and high welfare equilibria for tribes in violent competition with other tribes. This equilibrium allows for substantial investments in high-reward production—grain in the model. Furthermore, we show that such competition can decrease the required internal monitoring investments—due to a common-enemy effect. In contrast, chiefdoms are strictly welfare decreasing from the perspective of most of its participants. To put it sharply, a chiefdom is a social system wherein a coalition of bandits manage to coerce a number of other players to work for them—this in contrast to a large literature wherein chiefs usefully solve collective action problems (e.g., Olson, 1993; Konrad and Skaperdas, 2012; Dal Bó et al., 2022). Farmers might therefore view the collapse of their chiefdom as liberation from the taxing power of the dominant coalition. This potential animosity contributes to the lack of resilience of chiefdoms. Combined, these findings may go a long way in explaining the protracted emergence of enduring hierarchical forms of societies.

Our model thus differs markedly from the prominent models of hierarchy where the rise of hierarchy and a monopoly of violence is prosperity enhancing. There is a large strand of literature that investigates the function of ‘hierarchy’ or ‘government’ based on Olson-like models where situation of hierarchy is compared to anarchy

(Moselle and Polak, 2001; Grossman, 2002; Konrad and Skaperdas, 2012). Hence, an entrepreneurial player can install himself as 'stationary bandit', offering protection in exchange for taxation to other players. This leaves all players better off and especially the stationary bandit himself. In these models, the monopoly of violence is a public good, but not in our model, where perhaps deterrence may be seen as a public good in tribes, but in chiefdoms the monopoly of violence is certainly not. Our model is thus much more in line with the recent theoretical and empirical literature underlining the negative effects of emerging forms of hierarchy, private property rights and states on the welfare of most of its participants (e.g., Scott, 2017; Bowles and Choi, 2019; Mayshar et al., 2022).

Since our model is already quite complex, and involves many moving parts, we choose not to add further details, as these would come at the cost of omitting clearly relevant aspects. We limit ourselves here to highlighting three possible extensions.

First, introducing marriage, reproduction and death into our model, would bring out the role of institutions that outlive people even more clearly. Marriage adds a whole new set of evolutionary considerations to our game. Polygamy is a natural implication of our model, where elite males capture a disproportional share of the females, see e.g. Henrich et al. (2010). It also links up with the development of institutional mechanisms by which leading strata of society, the chiefs in our model, limit access of their offspring to the elite positions, for instance by primordial systems, see Olson (1982).

Second, our results highlight the requirement that long-term rewards of maintaining the coalition discipline the coalition-members and, in addition, that the chief assumes an information and coordination monopoly to limit the potential of coordinated threats from his soldiers. We evaluate individual deviations but not coordinated ones. However, the chiefdom equilibrium clearly offers scope for concerted deviation on the part of farmers (overthrowing the tax system) or soldiers (abusing the richer but individually weak chief). Given our equilibrium, we may expect permanent threats of 'palace revolts' within the coalition and, consequently, that the newly emerging hierarchical societies initiate further institutionalization of power, and demarcation of rights, roles, and rewards to limit the risk of instability of the dominant coalition (see also: North et al., 2009; Myerson, 2015; van Besouw et al., 2016; Bowles and Choi, 2019; Naidu et al., 2021). In our model, we have no politics or revolutions with a coalition of several people simultaneously deviating, thereby putting the dominant elite under pressure, as when a group of players suddenly solve the collective action problem for some short, revolutionary moment (Acemoglu and Robinson, 2006, 193–203). Extending the model to include this, would contribute to theories of institutional dynamics that we have been unable to develop here. This would require a more dynamic equilibrium concept to allow for evolving institutions and, in addition, for the possibilities of accumulation and dispersal of fortunes over time—see for attempts to offer a dynamic interpretation of elite coalitions, e.g., Grossman and Kim (1996); North et al. (2009); van Bavel et al. (2017); and Acemoglu, Robinson, and Torvik (2020).

Third, our model solves relevant commitment problems between soldiers and the chief using the sequence of phases within a stage of the game. Even though none of the actions of a player are enforceable by other players, the sequence of phases implies that other players can retaliate to deviations. Nonetheless, there are clearly more efficient solutions to commitment problems possible. For instance, the model features a head tax rather than a tax rate. Under the assumptions in our model, a poll tax is the most efficient system of surplus extraction. Adding further information restrictions on the investment of farmers in growing grain would make a tax rate the more efficient institution. However, this would increase the commitment problems of the dominant elite, since they would be tempted to raise taxes ex post, after the return on investment becomes visible (Milgrom et al., 1990; Greif et al., 1994). Similar, but historically more pressing, considerations are relevant for the interactions among the chief and his soldiers or other violence specialists (North et al., 2009).

Our model might also hold relevance to subsequent institutional transitions that have occurred further down the road of history. Our model shows why chiefdoms can be much larger than tribes by the efficiency gain achieved by the substitution of the mutual monitoring in the Single Tribe equilibrium by the information monopoly of the chief. However, though less pressing than in tribal societies, chiefdoms still face a size limit: the maximum number of agents a single person (i.e., the chief) can monitor. States, which started to emerge several millennia after the rise of chiefdoms, and often took their place, are structured differently, with a second level of bureaucrats (or governors), since the king can no longer monitor all individuals himself. However, this creates a new set of incentive problems: What motivates governors to report their private information truthfully to the king? What prevents one governor to rob from farmers who are entrusted by the king to other governors? How to avoid these governors using their discretionary power derived from their partial information monopoly to attack the king? Work addressing such questions can thus build on this paper.

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A Appendix

A.1 Constructive proof of Proposition 1

Let $\mathcal{N}_0$ be the full set of players and set $A_{i0} := A_i$ and set $n := 1$.

Step 1: separating out non-attacked players
Consider the subset $\mathcal{C}_n$ of $\mathcal{N}_{n-1}$ of players that are not attacked by another player from $\mathcal{N}_{n-1}$. Formally:

$$i \in \mathcal{C}_n \Longleftrightarrow \Sigma_{j\in\mathcal{N}_{n-1}} \mathcal{I}\left[I_{ji} = 1\right] = 0.$$

For this subset, the solution to equation (4) reads

$$Y_i = \max\left[A_{i,n-1}, 0\right]. \tag{42}$$

Step 2: netting out the attacks of non-attacked players on other players
For all players in the remaining subset $\mathcal{N}_n := \mathcal{N}_{n-1} - \mathcal{C}_n$, we net out the attack strength of attacks of non-attacked players. Formally:

$$A_{in} = A_{i,n-1} - \Sigma_{j\in\mathcal{C}_n} \mathcal{I}\left[I_{ji} = 1\right] A_{j,n-1},$$

for all $i \in \mathcal{N}_n$. A_{in} is i's attack strength net of the net attacks on i by $j \in \{\mathcal{C}_m, m = 1, n\}$.

Step 3: repeating this process for the remaining subset
Set $n := n + 1$ and repeat Step 1 and 2 until the remaining subset $\mathcal{N}_{n-1}$ contains no players that are not attacked by another player in this subset, or equivalently, until $\mathcal{C}_n = \varnothing$. Call this remaining subset $\mathcal{S} := \mathcal{N}_{n-1}$ and define $\overline{A}_i := A_{in}$. The remaining task is to calculate Y_i for all $i \in \mathcal{S}$.

Step 4: each member of $\mathcal{S}$ is attacked by exactly one other member of $\mathcal{S}$
By construction, $\mathcal{S}$ contains all players $i \in \mathcal{S}$ who are attacked by other member(s) $j \in \mathcal{S}$. Since $i \in \mathcal{S}$ may attack only one other player, this implies that i is attacked by exactly one $j \in \mathcal{S}$, not more than one. Formally:

$$i \in \mathcal{S} \Longleftrightarrow \Sigma_{j\in\mathcal{S}} \mathcal{I}\left[I_{ji} = 1\right] = 1.$$

Step 5: subdivision of $\mathcal{S}$ in the largest number of independent subsets
We subdivide $\mathcal{S}$ in the largest possible number M of subsets $\mathcal{S}_m$, $m = 1, M$, such that no member $i \in \mathcal{S}_m$ attacks a member of another subset $\mathcal{S}_k$, $k \neq m$. Such a subset $\mathcal{S}_m$ can be constructed by following the pattern of attacks: start from i, seek player j who is attacked by i (hence: $I_{ij} = 1$, a unique $j \in \mathcal{S}$ exists by Step 4), continue from

j, seek k who is attacked by j ($I_{jk} = 1$) and so on, and so forth, until you arrive at a player n who attacks i ($I_{ni} = 1$), establishing a circular pattern of attacks. By Step 4, at least one circular pattern exists, but $\mathcal{S}$ may also fall apart in multiple circles $\mathcal{S}_m$. Below we discuss a procedure to calculate Y_i's for each $\mathcal{S}_m$ separately. For the sake of convenience, we drop the subscript m in what follows.

Step 6: reordering players following the circular pattern of attacks
Let I be the number of players in $\mathcal{S}$. Without loss of generality, we can reorder players by redefining the index i of each player such that i attacks $i-1$ and is attacked by $i+1$; I is attacked by 1, completing the circle. Formally:

$$I_{i,i-1} = 1 \text{ for } i = 2, I, \qquad I_{1I} = 1.$$

For all $i \in \mathcal{S}$, condition (4) can now be simplified to:

$$Y_i = \max\left[\overline{A}_i - Y_{i+1}, 0\right] \Rightarrow \overline{A}_i \geq Y_i \geq 0, \tag{43}$$

defining $Y_{I+1} \equiv Y_1$ to close the circle. Compared to equation (4), this reformulation has eliminated the unknowns X_i. Apart from the non-linearity in the max function, (43) is a system of linear equations that has a unique solution since the determinant of the system is non-zero.

Step 7: eliminating the special case with equally armed players
Suppose $\overline{A}_i = \overline{A}$ for all $i \in \mathcal{S}$. Then, the solution to (4) for all $i \in \mathcal{S}$ reads:

$$X_i = Y_i = \overline{A}/2.$$

This gives the solution for all Y_i, which completes the proof.

Step 8: defining upper and lower bounds of Y_i
Set $n := 1$. Set $Y_{i0}^{\max} := \overline{A}_i$ and $Y_{i0}^{\min} = 0$ and

$$Y_{in}^{\min} := \max\left[\overline{A}_i - Y_{i+1,n-1}^{\max}, 0\right], \qquad Y_{in}^{\max} := \max\left[\overline{A}_i - Y_{i+1,n}^{\min}, 0\right]$$

for all $i \in \mathcal{S}$. By (4), this implies:

$$0 = Y_{i,n-1}^{\min} \leq Y_{in}^{\min} \leq Y_i \leq Y_{in}^{\max} \leq Y_{i,n-1}^{\max} = \overline{A}_i,$$

where strict inequality must apply for $0 = Y_{i,n-1}^{\min} < Y_{in}^{\min}$ for at least one i since $\overline{A}_i - \overline{A}_{i+1} > 0$ for at least one i, because $\Sigma_{i \in \mathcal{S}}\left(\overline{A}_i - \overline{A}_{i+1}\right) = 0$ due to the circular structure and the case $\overline{A}_i - \overline{A}_{i+1} = 0$ for all i is covered by Step 7. Similarly, strict inequality must apply for $Y_{in}^{\max} < Y_{i,n-1}^{\max} = \overline{A}_i$ for at least one i since $0 = Y_{i,n-1}^{\min} < Y_{in}^{\min}$ for at least one i.

Step 9: iterative narrowing of these upper and lower bounds
Set $n := n + 1$. Set

$$Y_{in}^{\min} := \max\left[\overline{A}_i - Y_{i+1,n-1}^{\max}, 0\right], \qquad Y_{in}^{\max} := \max\left[\overline{A}_i - Y_{i+1,n}^{\min}, 0\right] \tag{44}$$

Since $Y_{i+1,n-1}^{\max} \geq Y_{i+1}$, $Y_{in}^{\min} \leq Y_i$. Since $Y_{i+1,n}^{\min} \leq Y_i$, $Y_{in}^{\max} \geq Y_i$. Since $Y_{i+1,n-1}^{\max} \leq Y_{i+1,n-2}^{\max}$, $Y_{in}^{\min} \geq Y_{i,n-1}^{\min}$. Since $Y_{i+1,n}^{\min} \geq Y_{i+1,n-1}^{\min}$, $Y_{in}^{\max} \leq Y_{i,n-1}^{\max}$. Hence:

$$0 \leq Y_{i,n-1}^{\min} \leq Y_{in}^{\min} \leq Y_i \leq Y_{in}^{\max} \leq Y_{i,n-1}^{\max} \leq \overline{A}_i. \tag{45}$$

Step 10: checking for a solution
If for any i, either $Y_{in}^{\max} = 0$ or $Y_{in}^{\min} = \overline{A}_i$, then (45) implies that $Y_{in}^{\min} = Y_i = Y_{in}^{\max}$. Then Y_{i-1} can be calculated from Y_i by (43), and so on for $i-2$, $i-3$, until Y_i has been calculated for all i, which completes the proof.

If not, $Y_{in}^{\max} > 0$ for all i. Hence, (44) simplifies to:

$$Y_{in}^{\max} = \overline{A}_i - \overline{A}_{i+1} + Y_{i+2,n-1}^{\max}, \qquad Y_{in}^{\min} = \overline{A}_i - \overline{A}_{i+1} + Y_{i+2,n-1}^{\min}.$$

If $Y_{in}^{\max} = Y_{i,n-1}^{\max}$ and $Y_{in}^{\min} = Y_{i,n-1}^{\min}$ for all i. Then, the equations for $Y_{in}^{\max}$ and $Y_{in}^{\min}$ are two identical systems of linear equations, both with a unique solution. Hence, both solutions must be identical: $Y_{in}^{\max} = Y_{in}^{\min} = Y_i$, which completes the proof.

If not, $Y_{in}^{\max} < Y_{i,n-1}^{\max}$ for at least one i and $Y_{in}^{\min} > Y_{i,n-1}^{\min}$ for at least one i, implying that the set of upper and lower bounds have been tightened. Then, seting $n := n + 1$ and repeating Step 9 will eventually lead to a solution for Y_i for all i, which completes the proof.■

A.2 Proof of Proposition 5

This section presents the proof of Proposition 5 and the WT-E in Section 6.1.

Since the equilibrium is MP-E it can proven by backward induction.
First, consider a member i of a tribe taking actions X in the current round.

Robbery Phase: with probability $\mathrm{P_{WT}}$ the other tribe takes actions Y implying that i is attacked by a member j of the other tribe. Since $A_{\mathrm{WT}Y} - A_{\mathrm{WT}X} = \gamma^{-1}$, (6) implies that i's full grain budget is robbed: $G_{\mathrm{WT}X}^{C} = 0$. With probability $1 - \mathrm{P_{WT}}$ the other tribe takes actions X and does not attack. Since all tribesmen have invested the same amount $A_{\mathrm{WT}X}$ in arms in the Investment Phase, a planned attack on a member of either the own or the other tribe fails. Hence, these actions are weakly dominated by self-defense. If another tribesman d deviates on either $A_{\mathrm{WT}X}$ or $M_{\mathrm{WT}X}$

in the Investment Phase, i observes this. Since attacking is free, planning to attack d weakly dominates self-defense.

Taxation and Wage Payment Phase: it is never optimal for i to transfer grain to other players.

Investment Phase: (12) implies $B_{\mathrm{WT}X} = \mathrm{P}_{\mathrm{WT}}/\beta$. Since any deviation of a deviant d on either $A_{\mathrm{WT}X}$ or $M_{\mathrm{WT}X}$ invokes an attack by all tribesmen in the Robbery Phase anyway, monitoring has no value added for d. Hence: $M_d = 0$. d has two options: to attack a fellow tribesman in the Robbery Phase by setting $A_d = \gamma^{-1} + (L-1)A_{\mathrm{WT}X}$ or not to attack at all by setting $A_d = 0$. In both cases, d's payoff depends on whether the other tribe chooses action set X or Y. If the other tribe chooses X, all his fellow tribesmen attack him with combined strength $(L-1)\,A_{\mathrm{WT}X}$. His remaining strength γ^{-1} enables him to rob a fellow tribesman successfully, yielding $G_{\mathrm{WT}X}$. If the other tribe chooses Y, his attack on his fellow tribesman also succeeds, since the attacks of the other fellow tribesmen on himself are canceled due to the Priority of Self-defense against the other tribe. The attack of a member of the other tribe on d himself is repelled since $A_d - A_{\mathrm{WT}Y} = (L-1)\,A_{\mathrm{WT}X}$. The only issue is that he takes only a share of the booty, because part of the booty is taken by the attacker from the other tribe. Hence d is never robbed and therefore sets $B_d = 0$. Accounting for the booty sharing with the member of the other tribe, we obtain:

$$\begin{aligned} U_{\mathrm{WT}X} &= (1-\mathrm{P}_{\mathrm{WT}})\left(1 - A_{\mathrm{WT}X} - \mu\frac{L-1}{N-1}\right) + \frac{1}{2}\beta^{-1}\mathrm{P}_{\mathrm{WT}}^2 \qquad (46)\\ &= (1-\mathrm{P}_{\mathrm{WT}})\left(1 - \frac{1}{2}\mu\right) + \frac{1}{2}\beta^{-1}\mathrm{P}_{\mathrm{WT}}^2 + \mathcal{O}\left(N^{-1}\right), \\ U_{Xd} &< G_d + G_{\mathrm{WT}X} = 2 - A_d - A_{\mathrm{WT}X} - B_{\mathrm{WT}X} - \mu\frac{L-1}{N-1}. \end{aligned}$$

MP-E requires $U_{Xd} \leq U_{\mathrm{WT}X}$. Using $A_d = \gamma^{-1} + LA_{\mathrm{WT}X}$ and (34) for $B_{\mathrm{WT}X}$, a sufficient condition for $A_{\mathrm{WT}X}$ reads:

$$A_{\mathrm{WT}X} = \frac{1 - \gamma^{-1} + \mathrm{P}_{\mathrm{WT}}\left(1 - \beta^{-1} - \mu\frac{L-1}{N-1}\right) - \frac{1}{2}\beta^{-1}\mathrm{P}_{\mathrm{WT}}^2}{\frac{1}{2}N + \mathrm{P}_{\mathrm{WT}}} = \mathcal{O}\left(N^{-1}\right).$$

A similar argument applies to the case that the deviant sets $A_d = 0$. Using (33) and (46) shows that $U_{\mathrm{WT}X} \geq U_{\mathrm{OO}}$ is implied by (32).

Next, consider a tribe taking actions Y.

Robbery Phase: since $A_{\mathrm{WT}Y} - A_{\mathrm{WT}X} = \gamma^{-1}$, attacking a member of the other tribe yields a booty $G_{\mathrm{WT}X}$ if the other tribe plays action X in the current round; attacking a member of the other tribe dominates attacking a member of the own tribe, since the latter are better armed. If the other tribe plays Y, attackers are equally armed and hence there is no robbery.

Taxation and Wage Payment Phase: it is never optimal for i to transfer grain to other players.

Investment Phase: monitoring his fellow tribesmen tribe is not a profitable deviation since it does not yield a payoff. A tribesman who invests $A_{\mathrm{WT}Y}$ will never be robbed along the equilibrium path, irrespective whether the other tribe chooses X or Y; hence $B_{\mathrm{WT}Y} = 0$. The critical deviation of a deviant d is not investing in arms but in grain and tubers instead. Since $M_{\mathrm{WT}Y} = 0$, his fellow tribesmen will not observe this deviation and will therefore not respond to it. Since d is unarmed, he is fully robbed when the other tribe chooses Y. Hence, he sets $B_d = \mathrm{P}_{\mathrm{WT}}/\beta$. We obtain:

$$\begin{aligned} U_{\mathrm{WT}Y} &= 1 - A_{\mathrm{WT}Y} + (1 - \mathrm{P}_{\mathrm{WT}})\, G_{\mathrm{WT}X} \\ &= 1 - \gamma^{-1} + (1 - \mathrm{P}_{\mathrm{WT}}) \left(1 - \mathrm{P}_{\mathrm{WT}}/\beta - \frac{1}{2}\mu\right) + \mathcal{O}\left(N^{-1}\right), \\ U_{Yd} &= 1 - \mathrm{P}_{\mathrm{WT}} + \frac{1}{2}\beta^{-1}\mathrm{P}_{\mathrm{WT}}^2. \end{aligned} \tag{47}$$

MP-E requires $U_{\mathrm{WT}Y} = \max\left[U_{\mathrm{WT}X}, U_{Yd}\right]$. In a standard mixed strategy equilibrium, b must be indifferent between choosing Y or X, so $U_{\mathrm{WT}Y} = U_{\mathrm{WT}X}$. Here, b might be constrained in choosing Y by the incentive constaint $U_{\mathrm{WT}Y} \geq U_{Yd}$. Since (46) and (47) imply $U_{Yd} > U_{\mathrm{WT}X}$, we have $U_{\mathrm{WT}Y} = U_{Yd}$.

Solving $U_{\mathrm{WT}Y} = U_{Yd}$ for P_{WT} yields (33). For a real solution, the discriminant must be positive, which yields (30). Hence $\mathrm{P}_{\mathrm{WT}} \in (0, 1)$, as required for a probability. An interior solution for $B_{\mathrm{WT}X} \in (0, 1)$ imposes a stricter constraint: $\mathrm{P}_{\mathrm{WT}} \in (0, \beta)$. Even this stricter constraint is non-binding, since if $\mathrm{P}_{\mathrm{WT}} = \beta$, then $U_{\mathrm{WT}X} = 1 - \frac{1}{2}\beta - \frac{1}{2}(1 - \beta)\,\mu + \mathcal{O}\left(N^{-1}\right) < U_{\mathrm{OO}}$, which is ruled out by the constraint $U_{\mathrm{WT}X} \geq U_{\mathrm{OO}}$, which holds by (32). Since $G_{\mathrm{WT}X} = 1 - B_{\mathrm{WT}X} - \frac{1}{2}\mu + \mathcal{O}\left(N^{-1}\right) > \gamma^{-1}$ for robbery to pay off, $B_{\mathrm{WT}X} = \mathrm{P}_{\mathrm{WT}}/\beta < 1 - \gamma^{-1} - \frac{1}{2}\mu + \mathcal{O}\left(N^{-1}\right)$, using (34). Some simplification yields (31). The outside option constraint is automatically satisfied since $U_{\mathrm{WT}Y} > U_{\mathrm{WT}X} \geq U_{\mathrm{OO}}$.

The expected payoff reads

$$U_{\mathrm{WT}} = (1 - \mathrm{P}_{\mathrm{WT}})\, U_{\mathrm{WT}X} + \mathrm{P}_{\mathrm{WT}} U_{\mathrm{WT}Y}.$$

Using (46) and (47) and some simplification yields (36), which states expected utility as total endowment minus the costs of arms investment, monitoring, and efficiency loss of investment in tubers with the relevant probabilities.■

A.3 Proof of Proposition 7

This section presents the proof of Proposition 7 and the WC-E in Section 6.2.

Like the SC-E, the only point where the proof relies on the repeated game structure is in punishment of s-s who rob f-s unordered by c in Robbery Phase. In all other aspects, the equilibrium is a BP-E, which can be proven by backward induction, as we do below. We refer to the proof of Proposition 4 for issues that are identical to the SC-E.

Robbery Phase: t-s are unarmed, so cannot attack. f-s are ordered by c to attack a particular t. Any deviation yields them a lower payoff: c is well protected by 5 s-s, s-s are heavily armed, attacking another f fails as they are equally armed, and attacking another t forces them to share their booty with an f.

Wage Payment Phase: $W_{\mathrm{WC}cs}$ is motivated in the same way as in the SC-E.

Taxation Phase: Since $G^{W}_{\mathrm{WC}t} = G_{\mathrm{WC}t}$, the payoff of a f reads:

$$U_{\mathrm{WC}f} = 1 + A_{\mathrm{WC}f}\left(\gamma G_{\mathrm{WC}t} - 1\right) - T_{\mathrm{WC}fc} \geq U_{\mathrm{OO}}, \tag{48}$$

where the second term in the middle is the booty of the robbery minus the cost of arms. Using (40) for $G_{\mathrm{WC}t}$ and (8) for U_{OO} yields $T_{\mathrm{WC}fc} \leq \frac{1}{2}\beta$. c maximizes his payoff by making this constraint binding.

Investment Phase: Each t is attacked by a single f. Hence, t sets $B_{\mathrm{WC}t}$ according to (12):

$$B_{\mathrm{WC}t} = \frac{\gamma}{\beta} A_{\mathrm{WC}f} = \left(\gamma - 1\right)\gamma^{-1}.$$

$G_{\mathrm{WC}t}$ is set residually as $G_{\mathrm{WC}t} = 1 - B_{\mathrm{WC}t} = \gamma^{-1}$. Since $\gamma G_{\mathrm{WC}t} = 1$, investing in arms to defend themselves against fs' attack is weakly dominated by setting $A_{\mathrm{WC}t} = 0$. Consider a deviant tribesman d who invests in arms to rob an f. Since he robs successfully, he cannot be robbed himself. Hence, he sets $B_{\mathrm{WC}d} = 0$. Successfully attacking a f requires $A_{\mathrm{WC}d} = 2A_{\mathrm{WC}f} + \gamma^{-1}$: $2A_{\mathrm{WC}f}$, since the first $A_{\mathrm{WC}f}$ is needed to defend against the f who is attacking him, while the second $A_{\mathrm{WC}f}$ is needed to overpower the defense strength of the attacked f;[11] γ^{-1} is needed for the robbery. The booty robbed from f reads $G^{W}_{\mathrm{WC}f} = 1 - T_{\mathrm{WC}fc} - A_{\mathrm{WC}f}$. Hence, this deviation is weakly dominated by the equilibrium strategy if

$$U_d = 1 + T_{\mathrm{WC}fc} - 3A_{\mathrm{WC}f} - \gamma^{-1} \leq U_{\mathrm{WC}t} = 1 - A_{\mathrm{WC}f} - \frac{1}{2}\beta B^2_{\mathrm{WC}t}. \tag{49}$$

Substitution of the expression for $T_{\mathrm{WC}fc}$, $B_{\mathrm{WC}t}$, and $A_{\mathrm{WC}f}$ and some rearrangement yields (39), which holds by assumption. Attacking another t does not payoff since

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[11] The probability that t is attacked by the same f as he plans to attack is $\mathcal{O}\left(N^{-1}\right)$.

these are attacked each by a single f. Joining in that attack would require them to share the booty with the attacking f. Given that f's booty is just enough to cover the cost of $A_{\mathrm{WC}f}$ (since $\gamma G_{\mathrm{WC}t} = 1$), d's booty will not cover the cost of arms if he has to share the booty with f. Attacking a s fails, since these are heavily armed. Attacking c fails, since he is heavily defended by 5 s-s.

The payoff of c is a simple adaptation of (28). Since t-s do not invest in arms and monitoring, their payoff is equal to their initial endowment minus the cost of investing part of their endowment in tubers rather than grain and the cost of robbery by f-s. Since the latter robbery breaks even for f-s, its cost is equal to f-s' investment in arms. The payoffs for s and f are therefore the same as in the SC-E. The payoff of t equals $U_{\mathrm{WC}t} = 1 - A_{\mathrm{WC}f} - \frac{1}{2}\beta B^2_{\mathrm{WC}t}$.■